Residual stresses relaxation of welded structures under alternate loading

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ABSTRACT: As-built welded ship structures retain internal residual stresses after being manufactured that affect the structural performance of ship's components like plates, stiffened plates or more complex structures. The operation of the ship subjects the structure and its components to alternate loading that releases these residual stresses, total or partially, under certain conditions. The objective of this work is to analyse the stress relief process under in-plane alternate loading and to establish a formulation suitable for application on simplified methods of ship's structural analyses.

The residual stresses relaxation on unstiffened and stiffened plates is formulated and the correction to material stress-strain curves is presented for any stage of the relaxation process.

The balance of energy during the relaxation process is estimated in order to allow the evaluation of initial residual stresses level.

Finally it is presented the impact of residual stresses relief in the structural response of box girders under bending moment due to the application of several cycles of loading and unloading.

1 INTRODUCTION

During welding of marine structures on butt joints or stiffeners to plating connections, the material in the joint will be subject to a rapid heating followed by a rapid cooling period. This non-homogeneous temperature field will origin local plastic deformations and thus a residual stress field in the structure. Common for many weld geometries and welding situations is that this joining method will result in tensile residual stresses in the weld region, often with yield stress magnitude, and compression out of heat affected zone.

evaluation of the residual The stresses distribution along the structure has been concentrated in three main fields: development of non-destructive and destructive techniques (Okada et al., 2009), use of finite element methods and analytical studies associated with the mechanical characteristics of the different material and welding techniques.

Leggatt (2008) explains the residual stresses in welded structures, i.e. how the different factors affect the magnitude, directionality and distribution of residual stresses in welded joints and structures. There exist today several commercial FE-codes that may be employed for detailed nonlinear simulations of the development of the temperature and stress fields present during welding.

Lindgren (2001), Runesson et al (2003), Dong (2005) and Tekgoz et al. (2013) discussed methods for numerical simulation of the welding process taking into consideration actual thermal, mechanical and microstructure developments. The focus is on material modelling, coupling effects between thermal, mechanical fields and microstructure, numerical techniques and modelling aspects.

Recent studies of residual stress fields for specific welding methods include Tsirkas et al (2003) on laser welding and De Vuyst et al (2004) and Clerge et al (2003) on friction stir welding but they are of little applicability to steel ships structures.

The evaluation of the residual stresses level may be done by indirect way. It requires the imposition of some hypotheses to establish the residual stress pattern and may be obtained by two methods: the structural tangent modulus method and the total energy dissipation method (Gordo and Guedes Soares, 2004). The first one considers the variation of the tangent modulus at a point of a cycle corresponding to the maximum loading point in the previous cycle. The variation of the tangent modulus is the result of the variation on the effective inertia of the section due to the development of local plasticity at points where residual stresses are still high. The second method considers the dissipation of energy in a structure with residual stresses when an external load is applied. This method is further developed in this study.

2 RESIDUAL STRESSES MODELS OF AS-BUILT STRUCTURES

It is commonly accepted that welded marine structures presents a complex pattern of residual stresses due to welding of butt joints of adjacent plates, or corner T joints of the stiffeners or frames to the plating. However the structural analysis of structural components requires the adoption of models that represents the distribution of residual stresses in a simple and adequate manner.



Figure 1. Residual stress model for an unstiffened plate welded in the edges

Most of the research on this area considers a model for the plate with residual stresses that is represented by a strip on the edges and tops of the plate where the stresses are equal to the yield stress, σ_0 , in tension and the central part of the plate is in compression at a constant stress, σ_{rs} in the longitudinal direction or σ_{rst} in the transversal direction, as represented in Figure 1. The level of residual stress in this central region should ensure the equilibrium of the whole plate, according to the equations (1) and (2) for longitudinal and transversal equilibrium, respectively:

$$\sigma_o \cdot 2\eta t - \sigma_r \cdot (b - 2\eta t) = 0 \tag{1}$$

$$\sigma_o \cdot 2\eta t - \sigma_{rt} \cdot (a - 2\eta t) = 0 \tag{2}$$

Thus, the compressive residual stresses in each direction become:

$$\sigma_r = \sigma_o \cdot \frac{2\eta t}{b - 2\eta t} \tag{3}$$

$$\sigma_{rt} = \sigma_o \cdot \frac{2\eta t}{a - 2\eta t} \tag{4}$$

It is assumed that the width of tensile strip, ηt , is equal in both directions. In long plates where *a* is bigger than *b*, the transversal residual stresses are only a fraction of the longitudinal residual stresses and therefore they are normally neglected.

Also it is very common practice to neglect the effect of residual stresses in tension because the maximum stress of the plate is not affected. However, it becomes important for the study of structures under bending moment when relaxation of residual stresses occurs.

The direct effect of residual stresses on a perfect plate with free lateral edges is represented in Figure 2, presenting the average stress-strain curve of an elasto perfectly plastic material (path 'USOQR') and the correction for residual stresses effect (path 'OT' in tension and 'PR' in compression).



Figure 2. Average stress-strain curve for an unstiffened plate with and without residual stresses

The elastic perfectly plastic material behaviour is expressed by:

$$\phi_e = \begin{cases} -1 \iff \bar{\varepsilon} < -1 \\ \bar{\varepsilon} \iff -1 < \bar{\varepsilon} < 1 \\ +1 \iff \bar{\varepsilon} < 1 \end{cases}$$
(5)

where $\bar{\varepsilon} = \varepsilon/\varepsilon_o$ and $\phi_e = \sigma/\sigma_o$. Positive values correspond to compression and negative ones to tension.

Gordo and Guedes Soares (1993) consider two separate regions to account for the effect of residual stresses for the compression of the plate element: the edge strips under tension and the central region in compression. In the strips under tension the initial stress is the yield stress in tension and thus the strip behaves elastically until it reaches the yield stress in compression. This occurs when the average strain of the plate reaches twice the yield strain of the material.

The central region, initially in compression at a normalised residual stress of $\bar{\sigma}_r$, yields in compression at an average normalised strain of $1 - \bar{\varepsilon}_r$ where one has:

$$\begin{cases} \bar{\sigma}_r = \sigma_r / \sigma_o \\ \bar{\varepsilon}_r = \varepsilon_r / \varepsilon_o \end{cases}$$
(6)

The straight line between these two points, initiation of yielding in the central region (P) and yielding of the edge strips (Q), is expressed by:

$$\phi_r = \frac{\overline{\sigma_r} \cdot \overline{\varepsilon} + 1 - \overline{\sigma_r}}{1 + \overline{\sigma_r}} \tag{7}$$

It means that the edge strips alone support more load after the yielding of the central region until the swash load of the plate is reached. As consequence the structural tangent modulus of the plate is no longer E but:

$$E_{rc} = \frac{\overline{\sigma}_r}{1 + \overline{\sigma}_r} E \tag{8}$$

The situation is completely different in tension. One has some yielding of the edge strips from the initial point of loading because they cannot support any additional load since they are already at the yield stress. So the tensile load is strictly absorbed by the central region and the tangent modulus of the plate is therefore:

$$E_{rt} = E/(1+\bar{\sigma}_r) \tag{9}$$

The average normalised stress-strain curves of flat plates with residual stresses can be summarized by the equations below:

$$\begin{cases} \phi = \min(1, \ \bar{\varepsilon}, \ \phi_r) & \leftarrow \bar{\varepsilon} > 0 \\ \phi = \max(-1, \ \bar{\varepsilon}/(1 + \bar{\sigma}_r)) & \leftarrow \bar{\varepsilon} < 0 \end{cases}$$
(10)

where ϕ is the average stress normalised by the yield stress.

3 RELIEF OF RESIDUAL STRESSES DUE TO CYCLING LOADING

The structures are subjected to variation of loading during their life time. In particular, the ship's structure is under changing of axial and bending loading due to cargo, waves, movement of weight, etc.

Due to this variation of loading with time, residual stresses tend to be relieved by plastic deformations, changing the pattern of stress by achieving a different internal equilibrium.

For simplicity the analysis of this relief may be divided in three main cases: cycling loading predominantly in the tensile range, predominantly in compression or alternate loading in general.

3.1 Cycling load in tensile range

The loading of a plate at an average stress σ_a , point A of Figure 3, results in an average strain of $\bar{\varepsilon}_a = \bar{\sigma}_a(1 + \bar{\sigma}_r)$, which is the equal to the plastic deformation of the edges strips of the plate. During the discharge of load, the plate responds elastically through a new path, line AB, with a slope equal to the Young's modulus of the material.



Figure 3. Average stress-strain curve for an unstiffened plate with discharge of load

The equilibrium of the unloaded plate is achieved at point B with a residual strain that makes the plate longer and is given by:

$$\bar{\varepsilon}_{ra} = \bar{\sigma}_a \bar{\sigma}_r \tag{11}$$

The new residual stress pattern of this plate, after being subjected to the tensile stress $\bar{\sigma}_a$, is the one presented in point B (unloaded) with the following values:

$$\begin{cases} \bar{\sigma}_{rta} = -1 - \bar{\sigma}_a & (tensile \ strip) \\ \bar{\sigma}_{rca} = (1 + \bar{\sigma}_a)\bar{\sigma}_r & (central \ part) \end{cases}$$
(12)

Figure 4 presents graphically this reduction for an external stress equals to half of the yield stress.

The initial length of the plate, zero strain at point C, corresponds to the application of an average axial stress of:

$$\bar{\sigma}_{oa} = -\bar{\sigma}_a \bar{\sigma}_r \tag{13}$$

The stress-strain curve of the plate in compression, after being subject to initial tensile load, continues to be linear and elastic with a slope equal to the Young's modulus until the yielding in compression of the central part of the plate occurs. The corresponding stress is $1 - \bar{\sigma}_{rca}$ and the strain is $1 - \bar{\sigma}_{rca} + \bar{\varepsilon}_{ra}$.



Figure 4. Reduction of residual stresses after applying an external load, $\sigma_a = -0.5\sigma_o$.

If the cycling load is within this range then no further plastic deformation occurs and the relaxation process is only controlled by the maximum tensile load applied to the plate.

Equation (10) may be generalised to accommodate the effect of the residual stresses relief in tension as:

$$\phi = \max\left(-1, \frac{\bar{\varepsilon}}{1+\bar{\sigma}_r}, \bar{\varepsilon} - \bar{\sigma}_a \bar{\sigma}_r\right) \tag{14}$$

The validity of this formula is:

$$\begin{cases} \bar{\varepsilon} \le 1 - \bar{\sigma}_r \\ \bar{\sigma} \le 1 - \bar{\sigma}_r - \bar{\sigma}_a \bar{\sigma}_r \end{cases}$$
(15)

Thus the elastic limit in compression is achieved at the same strain of the initial plate but at a higher compressive stress because $-\bar{\sigma}_a\bar{\sigma}_r$ is positive. After this point one has plasticity in the central part of the plate.

Another important aspect is the energy's balance during alternate load of the plate because it may be used to estimate the level of residual stresses of a structure in experiments of plates or more complex structures.

The energy absorbed by plastic deformation during the tensile loading is the one that is dissipated in the tensile strips and it can be estimated as:

$$\mathbf{E}_{d} = -2\bar{\sigma}_{a}(1+\bar{\sigma}_{r})\cdot\sigma_{o}\varepsilon_{o}\cdot\eta t^{2}a \tag{16}$$

Part of this energy comes from the work done by the external load applied to the plate and the other part comes from the reduction of the internal potential elastic energy due to initial residual stresses relief.

The initial potential energy of the plate is:

$$\mathbf{E}_{po} = (1 + \bar{\sigma}_r) \cdot \sigma_o \varepsilon_o \cdot \eta t^2 a \tag{17}$$

The potential energy of the plate after the discharge of $\bar{\sigma}_a$ is:

$$\mathbf{E}_{pra} = (1 + \bar{\sigma}_a)^2 (1 + \bar{\sigma}_r) \cdot \sigma_o \varepsilon_o \cdot \eta t^2 a \tag{18}$$

The final balance of the potential energy is:

$$\Delta E_p = (1 - (1 + \bar{\sigma}_a)^2)(1 + \bar{\sigma}_r) \cdot \sigma_o \varepsilon_o \cdot \eta t^2 a \quad (19)$$

The external energy dissipated after applying the stress $\bar{\sigma}_a$ is:

$$\mathbf{E}_{dext} = \bar{\sigma}_a^{\ 2} (1 + \bar{\sigma}_r) \cdot \sigma_o \varepsilon_o \cdot \eta t^2 a \tag{20}$$

In the limit ($\bar{\sigma}_a = -1$) half of the dissipated energy comes from the work done by the external load and the other half comes from the initial potential energy.

It is also interesting to note that the ratio between the external energy dissipated and the total energy dissipated is proportional to one half of the applied stress, which means that initially the plastic energy is mainly coming from the reduction of the internal potential energy but, at high level of loading, the plastic work is essentially done by the external load.

3.2 Cycling load in compressive range

Applying a compressive stress $\bar{\sigma}_a$ greater than $1 - \bar{\varepsilon}_r$ to a plate with initial residual stresses as presented in Figure 1 will result in some plasticity in the central region and the behaviour of the plate is expressed by eq. (7). At the point D of Figure 3 the central region has a strain of:

$$\bar{\varepsilon}_a = \frac{\bar{\sigma}_a \cdot (1 + \bar{\sigma}_r) - 1 + \bar{\sigma}_r}{\bar{\sigma}_r} \tag{21}$$

according to eq. (7). In the discharge, the plate retains a plastic strain of:

$$\bar{\varepsilon}_{ap} = \bar{\varepsilon}_a - \bar{\varepsilon}_{ae} = \frac{\bar{\sigma}_a - 1 + \bar{\sigma}_r}{\bar{\sigma}_r} \tag{22}$$

where $\bar{\varepsilon}_{ae}$ is the elastic strain of the plate at a stress $\bar{\sigma}_{a}$.

The equilibrium of the unloaded plate is achieved at point E with the following residual stresses pattern:

$$\begin{cases} \bar{\sigma}_{rta} = \frac{\bar{\sigma}_a - 1}{\bar{\sigma}_r} & (tensile \ strip) \\ \bar{\sigma}_{rca} = 1 - \bar{\sigma}_a & (central \ part) \\ \bar{\varepsilon}_{ra} = \bar{\varepsilon}_{ap} = \frac{\bar{\sigma}_a - 1 + \bar{\sigma}_r}{\bar{\sigma}_r} \end{cases}$$
(23)

The condition of not having plastic deformations in the tensile edge strips defines the maximum average stress in tension in elastic domain $(-1 - \overline{\sigma}_{rta})$ and the corresponding strain is $-1 - \overline{\sigma}_{rta} + \overline{\varepsilon}_{ra}$ which coincides with zero strain of the initial plate, point F of Figure 3. After this point the edge strips yield in tension.

One has to note that the actual plate is marginally shorter than the initial one. In fact the actual length of the unloaded plate is:

$$l_a = l_o (1 - \bar{\varepsilon}_{ra} \cdot \varepsilon_o) \tag{24}$$

However there is no need for correction because the difference is of 0.12% for mild steel and less than 0.3% for high tensile steel plates.

Equation (10) may be generalised to accommodate the effect of the relief of residual stresses in compression as:

$$\phi = \max\left(-1, \bar{\varepsilon} - \frac{\bar{\sigma}_a - 1 + \bar{\sigma}_r}{\bar{\sigma}_r}, \phi_r\right) \leftarrow \begin{cases} \bar{\varepsilon} > 0\\ \bar{\sigma}_a \ge 1 - \bar{\sigma}_r \end{cases} (25)$$

The energy dissipated by plastic deformation during the compressive loading can be estimated as:

$$\mathbf{E}_{d} = \sigma_{o}(b - 2\eta t)t \cdot \left(\bar{\varepsilon}_{a} - (1 - \bar{\varepsilon}_{r})\right)\varepsilon_{o}a \tag{26}$$

and, for $\bar{\sigma}_a \ge 1 - \bar{\sigma}_r$, it leads to:

$$\mathbf{E}_{d} = 2 \cdot \frac{(\overline{\sigma}_{a} - 1 + \overline{\sigma}_{r})(1 + \overline{\sigma}_{r})}{\overline{\sigma}_{r}^{2}} \cdot \sigma_{o} \varepsilon_{o} \cdot \eta t^{2} a$$
(27)

The maximum elastic energy accumulated in the edge strip is:

$$E_o = \sigma_o \varepsilon_o \cdot \eta t^2 a \tag{28}$$

so eq.(27) becomes:

$$\mathbf{E}_{d} = 2 \cdot \frac{\overline{\sigma}_{a} - 1 + \overline{\sigma}_{r}}{\overline{\sigma}_{r}^{2}} (1 + \overline{\sigma}_{r}) \cdot E_{o}$$
⁽²⁹⁾

The potential energy of the plate after the discharge of $\overline{\sigma}_a$ is:

$$\mathbf{E}_{pra} = \left(\frac{1 - \overline{\sigma}_a}{\overline{\sigma}_r}\right)^2 \left(1 + \overline{\sigma}_r\right) \cdot E_o \tag{30}$$

The final balance of the potential energy is:

$$\Delta E_p = \left(1 - \left(\frac{1 - \overline{\sigma}_a}{\overline{\sigma}_r}\right)^2\right) (1 + \overline{\sigma}_r) \cdot E_o \tag{31}$$

The external energy dissipated after applying the stress $\bar{\sigma}_a$ is:

$$\mathbf{E}_{dext} = \left(2\bar{\sigma}_a - 1 + \left(\frac{1-\bar{\sigma}_a}{\bar{\sigma}_r}\right)^2\right)(1+\bar{\sigma}_r) \cdot E_o \qquad (32)$$

As previous, in the limit ($\bar{\sigma}_a = 1$), half of the dissipated energy comes from the work done by the

external load and the other half from the initial potential energy.

However, in compression, these formulas are only valid for $\bar{\sigma}_a \ge 1 - \bar{\sigma}_r$, the stress corresponding to the initiation of plasticity, as stated before.

3.2.1 Alternate loading in general

In the previous situations, predominant tension or compression, the final results depend only on the maximum applied stress in each case and the average stress-strain curves of the plate are unique.

In fact they cover the most common history of a plate belonging to a complex structure like a ship.

The consequences of the yielding of the edge strips are much more important than the yielding of the central part in compression and so, the tension cycle tends to dominate the overall process, especially when the first load cycle generates tension on the plate. To illustrate that, consider a plate with $\bar{\sigma}_r = 0.2$ and subjected to $\bar{\sigma}_a = -0.5$. After the application of $\bar{\sigma}_a$ the plate behaves elastically in compression until reaching an average stress of 0.9· σ_o according to eq. (15), which covers the most common loading histories.

But in general the final average stress-strain curve of a plate that suffers alternate plastic deformations in tension and compression is not unique and depends on the history of loading, i.e., the sequence of maximum stresses in tension and compression.

In terms of the response of a complex structure and its internal equilibrium, the most important aspect is the residual strain $\bar{\varepsilon}_{ra}$ that result from the load history.

The limiting values of the residual strain are $-\varepsilon_r$ and $+\varepsilon_o$. If tension is present in the initial stage of the load history, the residual strain is negative which corresponds to have a longer plate than the initial one in the end of the loading history.

4 APPLICATION ON 3-D THIN STRUCTURES UNDER BENDING MOMENTS

One of the most relevant alternate loading present in ship structures is the longitudinal bending moment which acts since the beginning of the ship's operation. Ship's hull is a stiffened walled structure where one of the most common arrangements is a set of stiffeners welded to plates having residual stresses due to fabrication.

In order to analyse the effects of the relief of residual stresses of plates in a 3-D structure it was chosen a closed box with rectangular cross section with a width B, and a depth D. The areas at bottom, side and deck are, respectively, A_b , A_s and A_d . A is the total area of the cross section. The bottom has *n* welding of stiffeners and each of them originates

two tensile strips with a width of the heat affected region of $\eta_o t$. So the panel has an equivalent η equal to $n\eta_o$ for structural evaluation.

Consider the application of a sagging bending moment to the box that creates a tensile stress $\bar{\sigma}_a$ in the bottom and a compressive stress in the deck less than $(1 - \bar{\sigma}_r)\sigma_o$ to avoid any plasticity in the deck.

After the removal of the bending moment the box should be in internal equilibrium but the bottom has a difference length according to eq. (11) and the correspondent remaining residual stresses are given by eq. (12). The average stress in the bottom for the initial length of the plate is given by eq. (13) and thus the box is not in internal equilibrium at zero strain.

The equilibrium is achieved by balancing the axial stress in x direction and the first moment of the stress distribution in respect to the neutral axis (Gordo and Guedes Soares (1996)) as follows:

$$\int \sigma_{ir}(z) \, dA = 0 \tag{33}$$

 $\int z \cdot \sigma_{ir}(z) \, dA = 0 \tag{34}$

where z is the vertical distance from the plate to the neutral axis.

The stress distribution has 3 components: the internal stress at zero strain, $\bar{\sigma}_{oa}$, an axial stress, $\bar{\sigma}_n$, to satisfy eq. (33) and a linear distribution of stress due to bending of the structure, $\bar{\sigma}_m$, to satisfy eq. (34), as shown in Figure 5.



Figure 5. Equilibrium of global internal stresses in a plate after relief of residual stresses in the bottom

Since the integral of $\bar{\sigma}_m$ over the cross section area is null, eq. (33) gives directly:

$$\overline{\sigma}_n = -\overline{\sigma}_{oa} \frac{A_b}{A} \tag{35}$$

The equilibrium of moments of the three components of the stress distribution relatively to the neutral axis located at a vertical position z_n to the base line leads to:

$$(\overline{\sigma}_{oa}A_b(-z_n) + \overline{\sigma}_n m_{eA})\sigma_o + M_m = 0$$

The static moment of the cross section area m_{eA} is null in respect to the neutral axis.

The moment M_m that counteracts the normalised stresses $\overline{\sigma}_{oa}$ and $\overline{\sigma}_n$ is:

$$M_m = z_n \overline{\sigma}_{oa} \sigma_o A_b \tag{36}$$

The distribution of stresses $\overline{\sigma}_m$ over the cross section that results from this bending moment is:

$$\overline{\sigma}_m = \frac{M_m}{I\sigma_o}(z - z_n) = \frac{z_n A_b \overline{\sigma}_{oa}}{I}(z - z_n)$$
(37)

The stress on the deck is then:

$$\overline{\sigma}_{deck} = \overline{\sigma}_{oa} \left(\frac{z_n A_b}{l} (z_{deck} - z_n) - \frac{A_b}{A} \right)$$
(38)

or:

$$\overline{\sigma}_{deck} = \overline{\sigma}_a \overline{\sigma}_r \frac{A_b}{A} \left(1 - \frac{z_n (z_{deck} - z_n)}{r^2} \right)$$
(39)

It means that the deck presents an average compression due to the global internal equilibrium after the box has been subjected to a sagging moment and subsequently discharged. A rough estimation of this compressive stress on ship's deck points for values of $\overline{\sigma}_{deck} = 0.1 \,\overline{\sigma}_a \overline{\sigma}_r$. This stress must be added to the residual stress pattern of the deck and changes the behaviour of the box and its ultimate moment and curvature.

The unloaded box retains in the end a residual curvature C_r given by:

$$C_r = \frac{M_m}{EI} = -\bar{\sigma}_a \frac{z_n A_b \bar{\sigma}_r \sigma_o}{EI}$$
(40)

This residual curvature has been reported in several experiments of box girders under pure bending.

In the presence of residual stresses in side shell the model may be extended to include the elongation in the part of the side that is below the neutral axis, as shown in Figure 6.



Figure 6. Equilibrium of global internal stresses in a plate after relief of residual stresses in the bottom and side.



Figure 7. Equilibrium of global internal stresses in a plate after alternate symmetric cycles.

Finally, in the case of progressive alternate symmetric moment that generates alternate axial stresses at the bottom and deck of $\pm \overline{\sigma}_a$ increasing in time, the third component of the stress $\overline{\sigma}_m$ tends to be null since one has the same effect in the deck and in the bottom on alternate cycles. The final result is a global distribution of internal stresses as presented in

Figure 7, and a reduction in the local initial residual stresses in the plate elements according to eq. (12).

The final curvature of the unloaded box in the case of a progressive increase of the alternate symmetric bending moment tends to minimal.

Experimental evidence of such effects due to residual stresses relief has been found in the tests of box girders. Gordo and Guedes Soares (2004) presented detailed results of the internal remaining strain after the discharge of a box girder subjected to initial cycles of a sagging moment. It is also reported the increase of the residual curvature with the increase of the maximum bending moment in each consecutive cycle.

Figure 8 presents the distribution of the strain's variation in the second cycle of loading. Initially all strain measurements were very close to zero, yellow line, but after been loaded until 10 mm of vertical displacement, the strains rise to the red line with an average value close to half of the yield strain and after discharge, black line, there is a global residual strain which is approximately 6% of the yield strain and 10% of the average applied strain.



Figure 8. Variation of strain in the bottom in a cycle of load and discharge of a box girder.

This elongation due to stress relief leads to a redistribution of residual stresses in the whole box and a residual curvature as predicted in eq. (40). Globally the redistribution of stresses in the unloaded structure creates a compressive state of stresses on the bottom panel, followed by a region under tension on the side plating that tends toward a slightly compressive stress on the top panel as predicted from eq. (39).

5 CONCLUSION

The loading of a plate or panel with residual stresses leads to a reduction of their level due to the occurrence of plasticity. This effect is more marked when applying tensile loads than when applying compression. A secondary aspect is the change of the plate's length after discharge, originating slightly longer plates after applying tensile loading and shorter plates after compression.

This last aspect has repercussion in 3D structures with residual stresses that have been previously subjected to a bending moment. The relief of residual stresses is concentrated in the regions under tension and, as a result, the structure retains a global internal remaining stress pattern that should be considered in addition to the local residual stresses. Also one may detect a residual curvature after the initial bending of the structure even for low levels of loading.

For cycling loads with increasing magnitude, the process is dominated by the maximum applied tensile stress at any stage and determines the distribution of remaining residual stresses of the structure.

The equations presented are adequate to be used in the description of the material behaviour for application in progressive collapse methods and FE models of complex structure like ship's hull. They also explain the experimental results on boxes under pure bending in respect to the global remaining residual stress pattern, residual curvature in each cycle and the change of the structural tangent modulus when the previous maximum stress is overlapped.

The balance of energy of a plate during a cycle of load and unload establishes that only a maximum of 50% of the dissipated energy comes from the work of the external load. The remaining part of the dissipated energy is obtained from the reduction of the potential elastic energy due to the residual stress relief. Such result may be applied in the estimation of initial residual stresses in experimental analysis of structures.

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