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COLLAPSE OF SHIP HULLS UNDER COMBINED VERTICAL AND HORIZONTAL BENDING MOMENTS

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ABSTRACT

The collapse of tanker hulls under combined vertical and horizontal bending moments is analyzed by an approximate method that accounts for the collapse strength of each stiffened plate element including in the post-collapse region. Four tankers are used in the analysis and the results are compared with interaction formulae.

INTRODUCTION

The improved knowledge of the collapse behaviour of plate elements as well as the generalisation of limit state design of ship structures has led to the development of various methods to predict the collapse load of the ship hull girders.

While the original formulation of this problem can be connected with Caldwell [1] who considered the collapse of the hull girder, including the effect of plate buckling, and to Faulkner [2] who proposed a simplified method to predict the collapse load of simple plate elements, several more recent proposals have dealt with various algorithms to achieve that aim.

Smith [3] was the first to propose a method to account for the behaviour the each individual element in the calculation of the ultimate behaviour of the hull girder. This was an hybrid procedure based mainly on a finite element formulation, but the plate behaviour was described by precalculated load deformation curves.

Several other authors have proposed alternative methods to perform that prediction. Billingsley [4], Adamchak [5], Rutherford and Caldwell [6] and Gordo, Guedes Soares and Faulkner [7] have chosen to develop simplified models of structural behaviour of the plate elements in order to construct the global moment curvature relation of the ship hull girder. Other authors have chosen a different line of work by developing simplified finite element formulations. Examples of such type of approaches are the contributions of Hori, Sekihama and Rashed [8], Yao and Nikolov [9], Paik [10], and of Bai, Bendiksen and Pedersen [11]. These simplified methods contrast with the heavy computational approach taken by Kutt, Piaszczyk, Chen and Liu [12], which proved not to be very practical for adoption in a design type of environment.

The method adopted in this work is based on a simplified formulation of the behaviour of plate-stiffener assemblies, described in Gordo and Guedes Soares [13]. The contribution of each element to the moment curvature relation of the ship hull was described in Gordo, Guedes Soares and Faulkner [7], and the predictions of this method were compared with various experimental results in Gordo and Guedes Soares [14], showing a very good correlation.

The work reported in those papers has considered the hull collapse under vertical bending moment, which is indeed the most important load effect in that context. However in many types of ships the combined effect of the vertical and the horizontal bending moments are important, and this work deals with the collapse of ship hulls under that combined load effect.

THE METHOD

Broadly speaking, the assessment of a moment-curvature relationship is obtained from the imposition of a sequence of increasing curvatures to the hull girder. For each curvature, the state of average strain of each beam-column element is determined. Entering with these values in the model that represents the load-shortening behavior of each element [13], the load that it sustains is calculated and consequently the bending moment resisted by the cross section is obtained from the summation of the contributions from the individual elements. The calculated set of values defines the desired moment-curvature relation.

Some problems arise in this implementation, because the discretisation of the sequence of the imposed curvatures strongly influences the convergence of the method due to the shift of the hull neutral axis. In this method, the modeling of the ship's cross section and the determination of the position of the neutral axis are important issues, as has already been pointed out in [14].

The basic assumptions of the method are the following:

- the elements are composed of longitudinal stiffeners and an effective breadth of the plate into which the cross section is subdivided and they are considered to act and behave independently,
- the ship cross sections are assumed to remain plane when during bending;
- the overall grillage collapse of the deck and bottom structures is avoided by using sufficiently strong transverse frames.

As a first step it is necessary to estimate the position of the neutral axis using an elastic analysis, because when the curvature is small the section behaves in the elastic domain. If the section is symmetric and the origin of the reference system is located on the baseline,(see fig. 1), the elastic neutral axis passes through a point with coordinates:

$$x_n = 0 \quad \text{and} \quad y_n = \frac{\sum y_i A_i}{\sum A_i} \tag{1}$$

where A_i and y_i are respectively the area and the vertical position of the stiffened element under consideration.

The most general case corresponds to that in which the ship is subjected to curvature in the x and y directions, respectively denoted as C_x and C_y. The overall curvature C is related to these two components by:

$$C = \sqrt{C_x^2 + C_y^2} \tag{2}$$

or:

$$C_x = C \cdot \cos\theta \quad \text{and} \quad C_y = C \ \sin\theta \tag{3}$$

adopting the right-hand rule, where θ is the angle between the neutral axis and the x axis. The strain at the centroid of an element i is ε_i which depends on its position and on the hull curvature, as given by:

$$\varepsilon_i = y_{gi} \cdot C_x - x_{gi} \cdot C_y \tag{4}$$

or substituting 3 in 4:

$$e_i = C\,(y_{gi} \cdot \cos\theta - x_{gi} \cdot \sin\theta) \tag{5}$$

where (x_{gi}, y_{gi}) are the coordinates of the centroid of the element i (stiffener and associated effective plate) referred to the point of intersection of the neutral axis and the center line. The relation between these local coordinates and the global coordinates are:

$$x_{gi} = x_i - x_n \quad \text{and} \quad y_{gi} = y_i - y_n \tag{6}$$

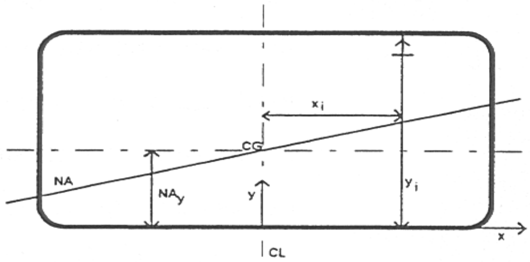

Figure 1
Combined bending of the hull girder

Equations (6) are still valid if one uses any point belonging to the neutral axis instead of the point used before.

Once the state of strain in each element is determined, the correspondent average stresses may be calculated according to the method described in [13] and consequently the components of the bending moment for a curvature C are given by:

$$M_x = \sum y_{gi} \cdot \Phi\,(\varepsilon_i) \cdot \sigma_o A_i \quad \text{and} \quad M_y = \sum x_{gi} \cdot \Phi\,(\varepsilon_i) \cdot \sigma_o A_i \tag{7}$$

where x_{gi} and y_{gi} are the distances from the element i to a local axes of a reference system located in the precise position of the instantaneous "center of gravity" (CG), and $\Phi(\varepsilon_i)$ represents the non-dimensional strength of the element, which has an appearance like the examples in fig.2.

The modulus of the total bending moment is:

$$M = \sqrt{M_x^2 + M_y^2} \tag{8}$$

This is the bending moment on the cross section if the assumed instantaneous position of the center of gravity is correct. However, during the stepwise process of increasing the hull's curvature, the location of the center of gravity is shifting and it becomes necessary to calculate the shift between two imposed curvatures. Rutherford and Caldwell [6] suggested that the shift could be taken equal to:

$$\Delta NA = \frac{\sum (A_i \cdot \sigma_i)}{C \cdot \sum (A_i \cdot E_i)} \tag{9}$$

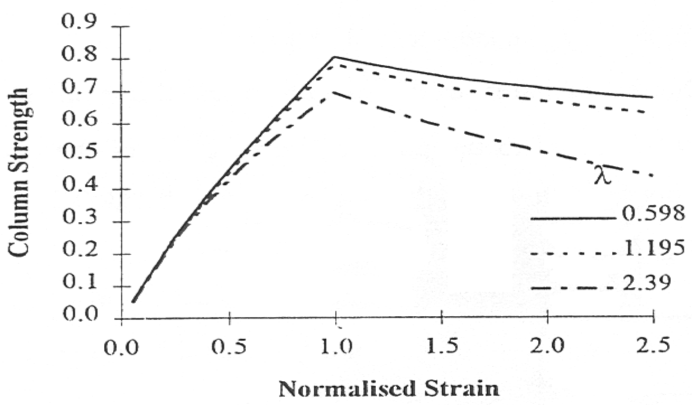

Figure 2

Load shortening curves of stiffened plates with plate slenderness of 2.32 and different column slendernesses, λ.

but, in this work, it was felt that this expression underestimated the shift and may cause problems in convergence.

For this reason a trial-and-error process was implemented, having as stopping criteria one of the two conditions: the total net load in the section, NL, or the error in the shift estimation ΔNA should be less than or equal to sufficiently low values. Equations (10) and (11) represent analytically these two conditions, where ξ was taken equal to 10^{-6}.

$$NL = \sum(\sigma_i \cdot A_i) \le \xi \cdot \sigma_o \cdot \sum(A_i) \qquad (10)$$

$$\Delta NA = k_E \cdot \frac{NL}{C \cdot E \cdot \sum A_i} \le 0.0001 \qquad (11)$$

The factor k_E is a function of the curvature and yield strain introduced to allow a better convergence of the method, and it is a result of the variation in the structural tangent modulus of the overall section with curvature.

The plate panels are treated according to the Faulkner's method for the flexural buckling of panels and the tripping of the stiffeners is estimated when necessary [13]. Different shedding patterns after buckling are available depending whenever flexural buckling or tripping is dominant.

VERTICAL AND HORIZONTAL ULTIMATE BENDING STRENGTH

The method described has been applied to assess the collapse of hull girders under vertical bending moments in [13] However the method is a general one capable of dealing with the combined effect of vertical and horizontal bending moments. This is achieved by imposing to the hull a curvature in the two orthogonal directions as indicated in eq. (2) and (3).

The application of this method will be exemplified here with the application to four tankers. The tankers considered in this study include three existing tankers and one VLCC that has failed under hogging in the harbour [6]. Their particulars are presented in Table 1.

Table 1

Particulars of the tankers

Name	Year	LBPP (m)	B (m)	P (m)	T (m)	DWT (t)	Cb
S. Mamede	1973	133.40	18.00	9.75	7.60	10250	0.7
Cercal	1979	230.00	42.00	19.80	12.70	80000	0.818
Bornes	1988	236.00	42.00	19.20	13.05	88900	0.805
Energy Conc.	1970	313.00	48.19	25.20	19.60	216269	

The ultimate longitudinal strength of these ships are summarized in Table 2 as well as the moment that corresponds to the first yield when the section is considered to behave elastically, denoted as yield moment, and the fully plastic moment without considering any buckling effects or shedding after yielding, denoted as plastic moment. The results of Table 2 are calculated considering the existence of 'hard-corners' in the intersection of the main framing and the plating and also at the intersection of the shear strake and the deck plating.

Table 2

Longitudinal bending moment of the tankers

Bending Moment	S.Mamede	Bornes	Cercal	Energy
Yield (MN.m)	0.980	8.161	8.259	19.332
Plastic (MN.m)	1.161	9.716	9.768	22.618
Sagging (MN.m)	0.910	7.123	6.652	16.392
Hogging (MN.m)	0.932	8.354	7.120	19.164
Horizontal (MN.m)	1.300	11.844	9.857	22.479
Sagging/Yield (%)	92.9	87.3	80.5	84.8
Hogging/Yield (%)	95.1	102.4	86.2	99.1
Plastic/Yield (%)	118.5	119.1	118.3	116.3
Hogging/Sagging (%)	102.4	117.3	107.4	116.9
Horizontal/Sagging (%)	142.9	166.3	148.2	137.1

Some general conclusions for these type of ships may be readily recognized. The form coefficient between the plastic and yield moment is approximately 1.18 for single skin tankers and its variability is low. However these ships are old designs and one may expect the increase of this coefficient in double skin tankers.

The ultimate bending strength of the tankers in sagging are always below the yield moment by a difference that may be as large as 20%. The reduction of the ultimate strength seems to increase with the length of the ship and it is directly related to the increase of column slenderness of the stiffener plate elements with the increase of the length. The limitation of the maximum frame spacing is one way of minimising this tendency.

The ultimate bending moment in hogging is normally of the same magnitude of the first yield moment, but a difference of 14% is found in one of the ships. The ratio between horizontal and sagging bending moment is normally higher than 1.4 and this ratio is of especial interest for the analysis of the combined bending.

COMBINED BENDING

The behavior of the ships under combined vertical and horizontal bending moment presents some particular aspects that result from the usual geometry of the ships, be it the overall geometry of the hull or the dependence on location of the geometry and dimensions of the stiffeners and plating. This happens because the plate and stiffener slendernesses of the stiffened plate elements of the shell are normally different from those of the deck and bottom. The shell plating governs the horizontal component of the collapse moment while the deck and bottom are more related with the vertical component. Moreover, the typical distances to the neutral axis of these panels are quite different and, consequently, the strain in a plate can be associated with substancially different hull curvatures depending on its position. The inelastic effects may also play an important rule.

Thus, it is normal to find in the analysis of any section that the angle between the moment vector and the neutral axis is changing during the load process if the direction of the latter is kept constant. Also the minimum ultimate moment may not be achieved in the vertical position and the maximum carrying capacity of the section to sustain bending is obtained with a moment near but not exactly equal to the horizontal bending moment.

Figures 3 to 6 show how the relation between the ultimate moment and the angle that the neutral axis makes with the x axis. The components of the moment about the two principal axis are also plotted.

The model of the cross section of each ship is adjacent to the corresponding graphic. Each point represents a stiffened plate element and it is possible to identify at the top right corner the elements that have already buckled, represented by squares, immediately before the collapse of the whole section when the imposed curvature has an angle of 20° from pure sagging. This angle is approximately the angle at which the vertical and the horizontal moment have the same magnitude as may be seen in the graphics of the same figures where the intersection of the vertical and horizontal components of the moment vector is always located at an angle between 20 and 30°.

Surprisingly, the variation of the vertical component of the moment, M_x, is almost linear from sagging (0°) to hogging (180°) when one could have expected it to be sinusoidal. This last type of behavior is observed on the variation of the horizontal component, M_y. The function is, in this case, fuller than a sinusoid and very close to a parabola. Because of the linear variation of the vertical moment, some ships present minimum values for the ultimate moment lower than the sagging or hogging moment. However the difference of values is not high and thus, the ultimate moment may be considered constant at low angles around pure sagging or hogging.

Table 3 compares the minimum value of the ultimate bending moment at small angles of heel with the corresponding ultimate moment at the upright position for sagging and hogging. The differences are very low, but one may expect higher reductions if the cross sections are modeled without considering 'hard-corners'. In fact the real behaviour of the structure is somewhere between the models.

Table 3

	Sagging			Hogging		
Ship	Upright	Minimum	Angle	Upright	Minimum	Angle
S. Mamede	0.910	0.910	0°	0.932	0.925	10°
Cercal	6.652	6.649	5°	7.120	7.120	0°
Bornes	7.123	7.121	5°	8.354	8.237	10°
Energy	16.392	16.250	10°	19.164	18.029	20°

The interaction between vertical and horizontal moments are plotted in Figure 7. Several curves compare the calculated points for sagging and hogging with interaction curves that vary from linear to quadratic.

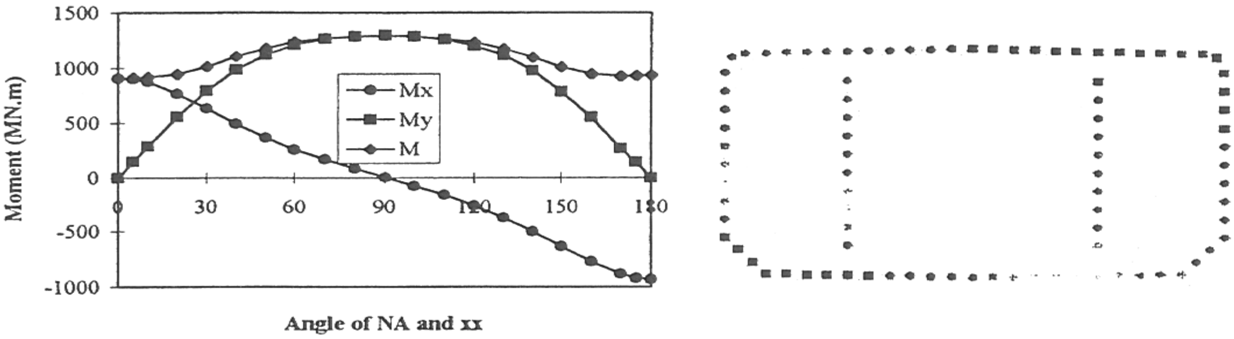

Figure 3 - S. MAMEDE

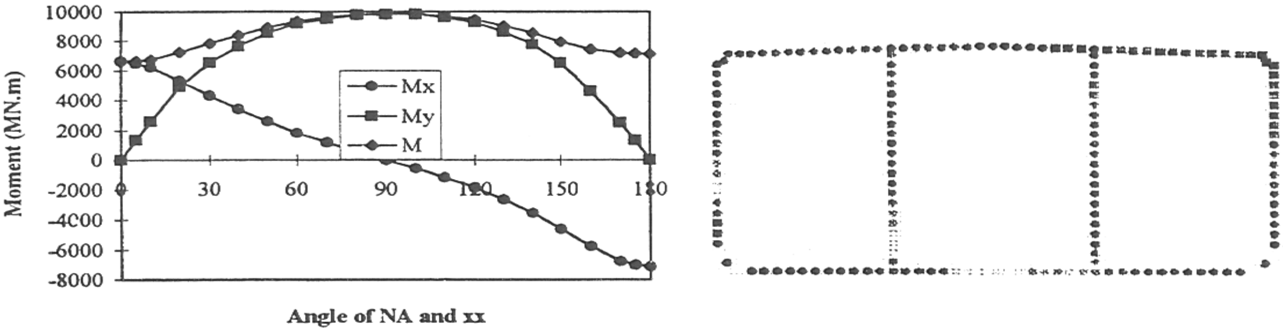

Figure 4 - CERCAL

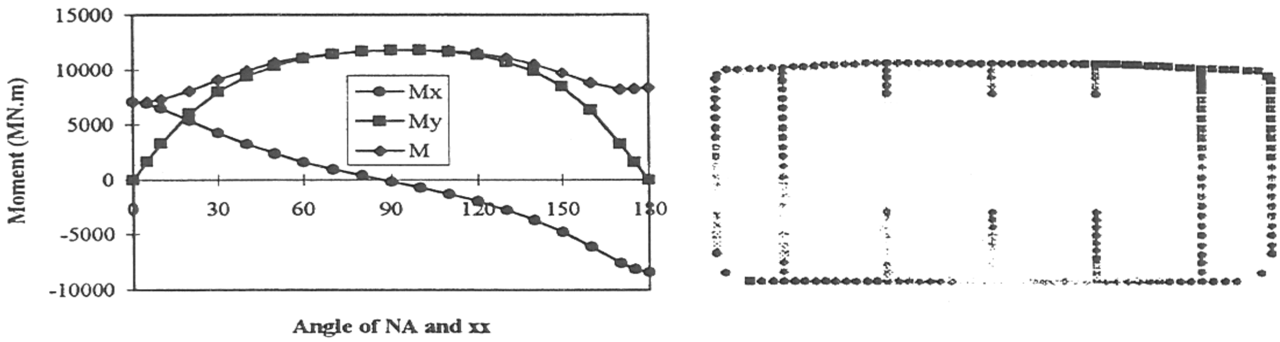

Figure 5 - BORNES

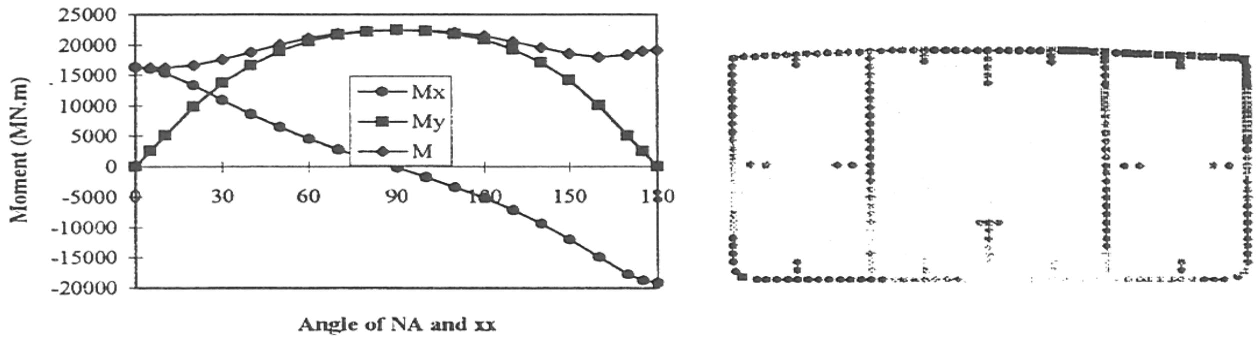

Figure 6 - ENERGY CONCENTRATION

The governing equation is given by:

$$\left(\frac{M_x}{M_{uv}}\right)^{\alpha} + \left(\frac{M_y}{M_{uh}}\right)^{\alpha} = 1 \tag{13}$$

where M_{uv} and M_{uh} are respectively the vertical and horizontal ultimate moment. The vertical ultimate moment may be the sagging or the hogging ultimate moment depending which is the quadrant under analysis. The parameter α is tentatively used on the graphics of Figure 7 with the values of 1.0, 1.5, 1.66 and 2.0.

Good interaction is achieved with α between 1.5 and 1.66 and it seems unnecessary to use different exponents in hogging and sagging for this type of ships.

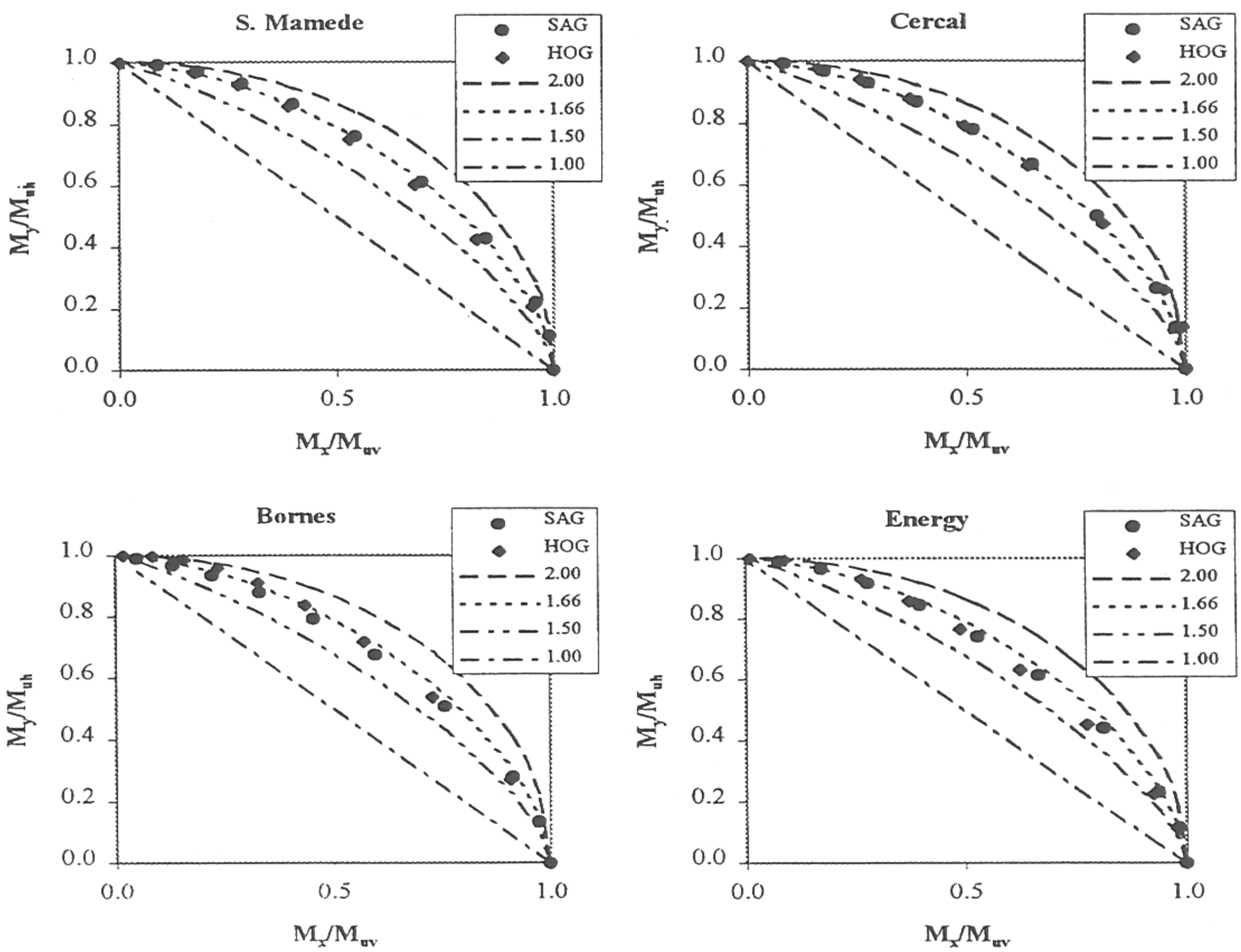

Figure 7

CONCLUSIONS

A method of calculating the ultimate collapse load of hull girders has been applied to the collapse of four tankers under combined vertical and horizontal bending moments. It was found that the ratio of plastic to elastic yield moment was about 1.18 for this type of topology. The results were compared with an interaction curve with a varying exponent. It was found that an exponent between 1.5 and 1.66 would be representative of the results obtained.

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