

A Simplified Method to Simulate Residual Stresses in Plates

J.M. Gordo & G. Teixeira

CENTEC, IST, University of Lisbon, Portugal

ABSTRACT: Welded structures are subjected to internal residual stress after manufacturing that may affect the structural strength and normally are associated with an increase on initial geometrical imperfections. This study presents a simplified method to generate an adequate representation of residual stresses on Finite Element models for structural analysis of thin-walled structures and other applications. The results obtained show that the methodology proposed to introduce residual stresses is simple, accurate and efficient on the modulation of post-welding stresses and their pattern, thus it may be used for simulation of the thermal process.

1 INTRODUCTION

Many efforts have been made to simulate the process of welding. Given that the process of welding is a phenomenon using metal in the liquid state, and as such without solid-state stresses, slightly above fusion temperature, the reality is quite difficult to emulate. This phenomenon is usually neglected, and the analysis is concentrated on the heat flux directed to the plate on the welded zone. The heat source, as such, can be of two types: fixed or moving. The first one to be described is the approximation proposed by Rosenthal (1946). The analytical theory of Rosenthal is based on a concentrated heat source. Later, Friedman (1975) presented the Gaussian Distributed heat source, as to express an approximation of the heat flux of the moving heat source. More recently however, a study (Goldak et al. 1985) proposed the usage of a Double-Ellipsoidal moving heat source.

Different heat inputs with different welding speeds have a direct effect on not only the results, but in the simulation and computation as well. Heinze et al. (2012) have studied this phenomenon, which compares different heat inputs with the results obtained.

The *FEA* of the welding process is often defined as a thermo-mechanical coupled analysis. Generally, there are three methods to simulate the welding process. The first method is the most straightforward of the three methods used. It consists in applying directly the temperature field to the plate, with its geometry already defined with structural elements, to calculate directly the distribution of residual stresses and the plate deformation. The temperatures are defined as body loads, and the plate is heated and cooled along time as such.

The two most used methods are known as the direct and indirect methods. The direct method, as the name implies, is characterized by the usage of elements that make possible both a thermal and structural analysis. The indirect method, which is the method more commonly used in simulations of this type, uses a semi-coupled approach. This approach predicts two different steps in the calculations, one in which the heating process is simulated, and elements which only have thermal degrees of freedom are used. This approach was used in studies from Chen (2011), Chen et al. (2011), and later Chen & Khameneh (2013) to study the deformations welding causes to specimens with 300 mm by 260 mm and 6 mm in thickness.

The geometry used is commonly accepted to be the three-dimensional in general for all simulations. However, it is especially difficult and time-consuming to compute 3D plates with initial imperfections. In these cases, it is common to use a 2D simulation.

Regarding mesh size, when performing an analysis of the deformation due to the heat of the welding process, it is common to use a more refined mesh in the area of the welding chord, and directly near it, as done in the study performed by Chen (2011). For a more refined simulation, the use of dynamic meshes can be applied. In general, dynamic meshing consists of having meshes solidary to the moving heat source. This saves computation time. Runesson & Skyttebol (2007) make a brief case of the methodology used to perform analysis of the welding process in their study.

In modelling the welding process, it is crucial to make the right choice regarding the boundary conditions of the model to be simulated. The boundary conditions affect drastically the outcome of the results. It

is of the biggest importance to have the boundary conditions in mind when performing the simulations. With that in mind, when it comes to choosing the boundary conditions, in terms of the structural analysis, it is common for the plates to be simply supported, clamped, or completely free.

A study has been made (Fu et al., 2014) to simulate this effect. The authors compared these boundary conditions to the other cases. It was found that the boundary conditions that restrained the plate have less strain but tend to have more residual stresses. For the clamped case, there are two methods in which the clamping can be simulated. In the study made by Deng (2015), the clamping is simulated by applying a friction coefficient and a force to specific nodes.

The simplest case is to lock the nodes in the areas in which the clamps are attached to simulate clamping, since the movement on the clamping is minimum and can usually be neglected. This approach was used in the study performed by Fu et al. (2014), with the results obtained being very acceptable.

2 RESIDUAL STRESSES APPROACHES

2.1 Stress-strain Curve Approximation

Gordo & Guedes Soares (1993) proposed a simplified approach to model analytically the distribution and values of the induced residual stresses. In this method, the authors considered a bilinear isotropic elastic-plastic approximation (*BISO*) to represent the material behaviour. This consideration is acceptable in the analysis of structures made of very ductile materials having a yielding plateau. The equation that models this is defined by branches and is the one seen in (1).

$$\phi_e = \begin{cases} -1 & \Leftarrow \bar{\epsilon} < -1 \\ \bar{\epsilon} & \Leftarrow -1 < \bar{\epsilon} < 1 \\ +1 & \Leftarrow \bar{\epsilon} > 1 \end{cases} \quad (1)$$

Where ϕ_e is the stress on the material and $\bar{\epsilon}$ is the strain normalised by yield strain. This equation considers both compression and tension of the material.

2.2 Residual Stress Distribution

During welding, the temperature between the sides of the plate, which maintain an even temperature along its width similar to the environment temperature, and the area near the welding, are severely different. According to Masubuchi (1980), the residual stresses that appear on a plate can be of two types:

1. Residual welding stresses that are produced directly by the thermal difference during the welding;
2. Residual stresses caused by external restraint.

Masubuchi & Martin (1965) devised an equation for the distribution of residual stresses in the longitudinal direction along the width of the plate, and is expressed as it can be seen on the equation (2).

$$\sigma_y(x) = \sigma_m \left[1 - \left(\frac{x}{w} \right)^2 \right] e^{-\frac{1}{2} \left(\frac{x}{w} \right)^2} \quad (2)$$

Where σ_y is the residual stress in the longitudinal direction, σ_m is the maximum stress and w is the width of the plate. This equation considers the area of the plate in traction, as well as the area in compression. The value for the σ_m can be as high as the yield stress. The distribution of residual given by equation (2) can be seen in Figure 1.

This distribution has a distinct point in which the stresses change from tensile to compressive. The value of η can be easily obtained by substituting the value for the longitudinal residual stress by zero in the equation and solve for x .

However, it is usual to consider a model, which simplifies the distribution of residual stresses along the width of the plate. With that, it is possible to define the residual stresses in tension as being located in a strip of width ηt , with η being the ratio between the plate's thickness t and the strips' width. The remaining $w - 2\eta t$ width of the plate is in compression.

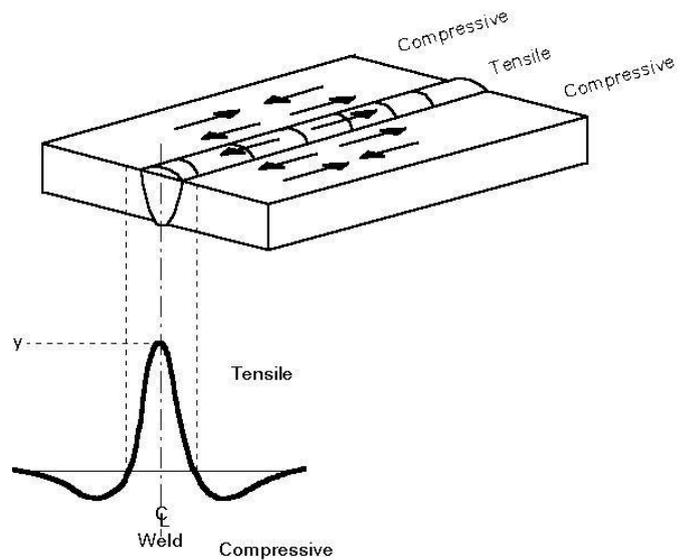


Figure 1 - Residual stresses' distribution parallel to the welding direction

For the value of η , Guedes Soares & Soreide (1983) state that, although commonly varying from 2 to 8, it is common even to consider values below that. As such, a value of 1.5 was set, and used along the calculations, as well as the geometry for the Finite Element Analysis (*FEA*). Since the model used considers two adjacent plates, the total width of the strip is equal to $2\eta t$. It is in this area that the residual stresses are significantly bigger. These are predominant in the

longitudinal direction, and their value can be obtained using equation (3).

$$\sigma_r = \sigma_o \frac{2\eta t}{w-2\eta t} \quad (3)$$

σ_r and σ_o are the resulting residual and yield stresses, respectively. Timoshenko & Goodier (1982) asserts that a plate experiencing a gradient of temperature along its width is bound to develop residual stresses. However, for simplicity purposes on performing calculations, a theoretical model has been created. This model features a constant distribution for both the traction residual stresses and compression residual stresses.

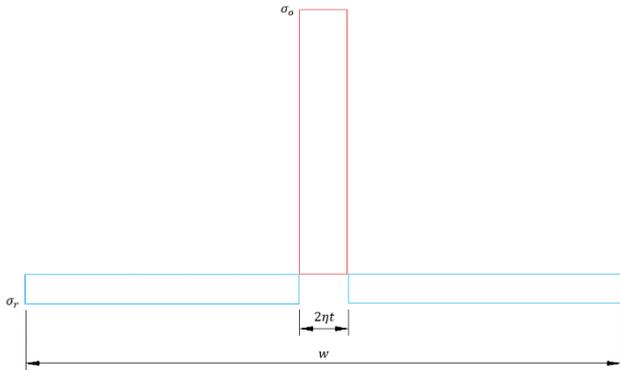


Figure 2 - Theoretical Model for the Distribution of Residual Stresses

3 MODELLING OF RESIDUAL STRESSES

The FEM model here implemented makes use of a two-dimensional (2D) geometry, where elements of shell type were used. In ANSYS®, elements of the type SHELL281 were used. A quadrangular mesh was used. The mesh is finer in the middle, where the welding toe is located, and coarser in the edges, where low refinement can be considered. The material behaviour is described by the Equation (1), where the Bilinear Isotropic (*BISO*) correction to the elasto-plastic domain was used, with no plastic hardening considered. This behaviour was considered for both traction and compression.

Regarding heat input, a time-dependent heat source was considered, which is uniform along the length of the welding path, to promote speed in the computational process. Various temperatures were considered. Finally, concerning the boundary conditions, two cases were implemented. The first one features a fixed node translation on the transversal direction, applied on the nodes located on the longer edges of the plates. This was considered to promote symmetry, as leaving one edge free would create asymmetry on the resulting geometry. For the first case the shorter edges were left free, whereas in the second case, the shorter edges are allowed only to move as a rigid line, as to simulate a welded plate on these shorter edges.

3.1 Temperature Interval

The simulation of the generation of residual stresses in a structure requires to input a temperature variation that creates the tensile residual stresses pattern in the welded region.

At high temperature in this region its material is in compression at yield stress for that temperature; during cooling, the region becomes to behave elastically, reducing the compressive state of stress and, when the reduction of temperature is enough, it becomes to be in tension. Tensile stress increases until the yield stress in tension is achieved. After that reduction in temperature and for elastic-perfectly plastic material the residual stresses pattern remains constant and only the plastic strain increases.

The minimum difference of temperature required to generate the pattern of residual stress at room temperature involves the knowledge of the yield stress and elastic modulus E at the 2 extreme temperature and the coefficient of thermal expansion, α .

A major of this minimum variation of temperature is given by:

$$\Delta T = -\frac{2\sigma_o}{\alpha E} \quad (3)$$

The steel used in this study was *ASTM 36* Structural Steel which has yield stress of 370 MPa at 20° C and $\alpha=11.3 \times 10^{-6} \text{ K}^{-1}$.

The temperature difference required is $\Delta T=-312^\circ$.

3.2 Implementation of the FEM

In order to validate the feasibility of the two-dimensional (2D) approach for the application of weld-induced residual stresses, a test model was devised.

The dimensions of the plates used were firstly 10 m by 5 m, and later 2 m by 0.8 m. Models with imperfections and with added stiffeners were created by modifying the existing geometry with 2 m by 0.8 m. The thicknesses used were three, of 6, 10 and 20 mm, as these are common thicknesses of plates used in the Naval Industry.

Since this model is to simulate a simplified distribution of residual stresses, the loading is made using only a direct heat input along the full length of the welding path. This heat input is also conducted in the full width of the area of the plate subjected to a traction stress. This width is equal to the tensile strip with $2\eta t$.

Finally, the boundary conditions chosen ensure that the plate has some freedom to move. However, in order to obtain symmetry transversally to the welding path, the longer edges were constrained on the transverse direction, in the x direction. Also, in a second approach, and to simulate the welding on the transversal edges, a constraint of rigid edge in the longitudinal direction was created. Both edges were left free to move.

Regarding the heat source, this was modelled by applying directly the temperature as a body force to the plate. The temperature input was performed using a time-dependent table. In Figure 3 can be seen the temperature distribution, varying as a function of time. As it can be seen in the figure below, the temperature increases to 500° Celsius in 500 seconds, with a decrease to the 20° Celsius.

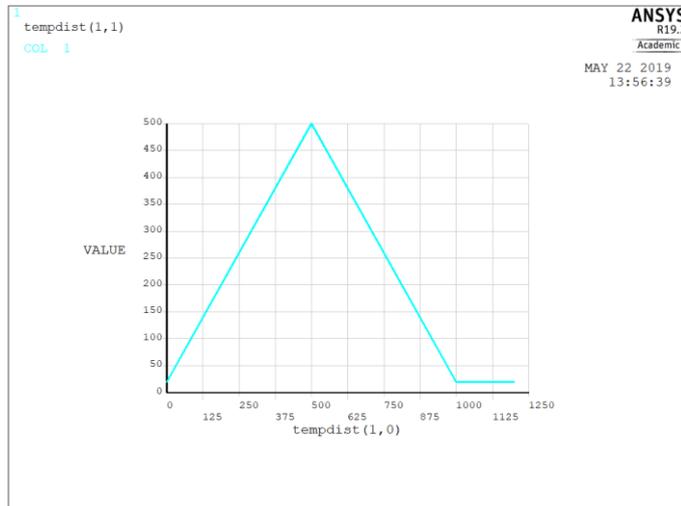


Figure 3 – Time-dependent temperature distribution

3.2.1 Butt weld of long plates

Figure 4 presents the mesh and the deformation of welded plate with 10 m by 5 m with $\eta=3$ and $t=10\text{mm}$. It was found during the calibration of the model that the mesh size should be smaller than $1/3$ of ηt in order to achieve a good representation of the residual stresses pattern. The resulting distribution of longitudinal residual stresses on the plate is presented in Figure 5.

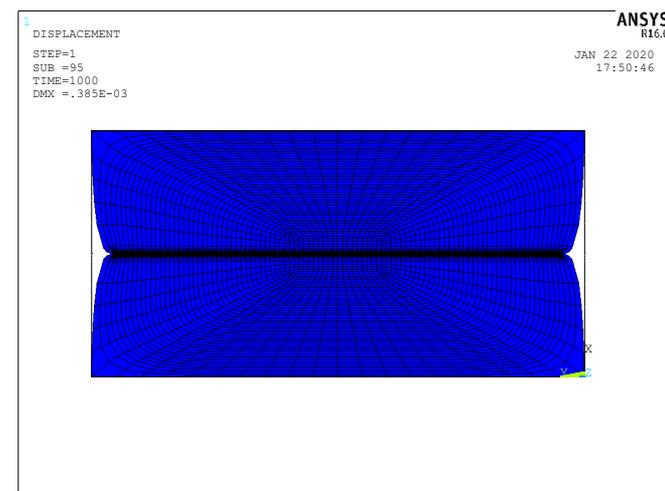


Figure 4 – Mesh and final deformation of a 10mm thick plate with 10x5m after butt welding.

This simulation of butt weld of 2 long plates shows the shrinkage at the ends of the welding and the residual stress pattern is uniform along the length except in the ends of the welding due to uncompensated thermal loads during the cooling. The level of compressive residual stresses in this butt weld is very low due

to the very high ratio between the compressive and tensile width of the plate.

In Figure 6 it is shown the distribution for the longitudinal residual stresses at mid-length of the plate near the welding. The tensile strip is subject to 373 MPa and the compressive region to less than -3 MPa due to the high ratio already mentioned.

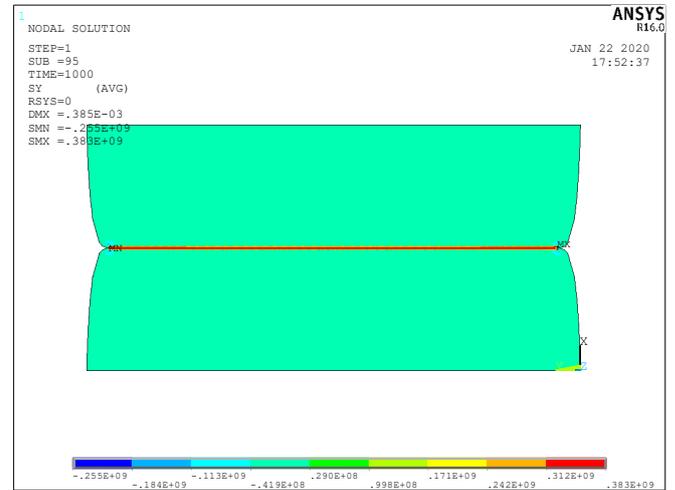


Figure 5 - Distribution of the residual stresses in the longitudinal direction for a long plate with 10 mm thickness

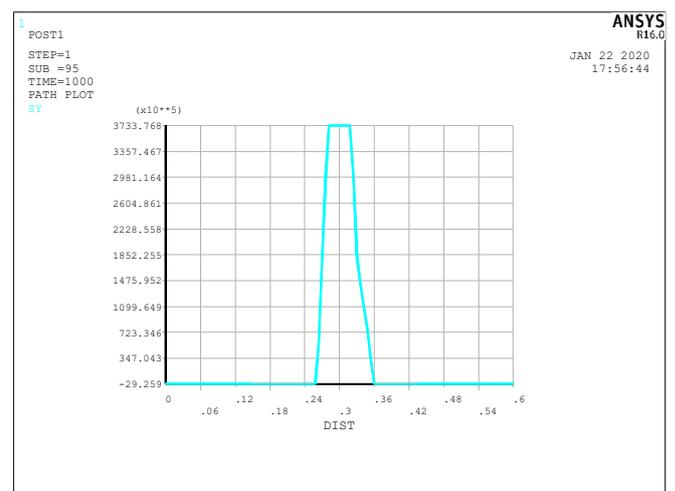


Figure 6 - Distribution of the residual stresses in the middle of a 10x5m plate with 10 mm thickness and $\eta=3$. Detail near the welding.

The pattern of compressive stresses changes with the longitudinal position as can be seen in Figure 7. In the middle compressive stresses are almost constant due to symmetry and the location been far away from the tops of the part. At half-quarter of the length of the plate the distribution shows an evident shear lag effect in result of unbalanced longitudinal stresses at the tops. The pattern of tensile stresses is very similar in all situations, represented by the almost vertical straight line.

In the middle of the weld the residual stresses are almost constant except near the tops, Figure 8. The tops create unbalanced forces in longitudinal direc-

tion that are transform in distortions and shear originating a complex pattern of stress, as shown in Figure 9.

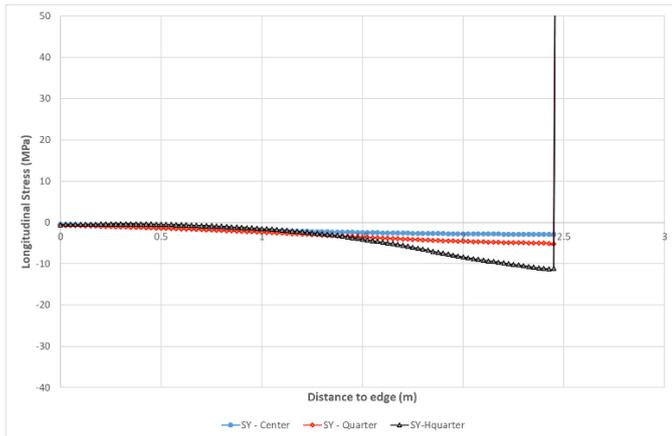


Figure 7 - Distribution of the compressive residual stresses in the middle, quarter and half-quarter of a 10x5m plate with 10 mm thickness and $\eta=3$.

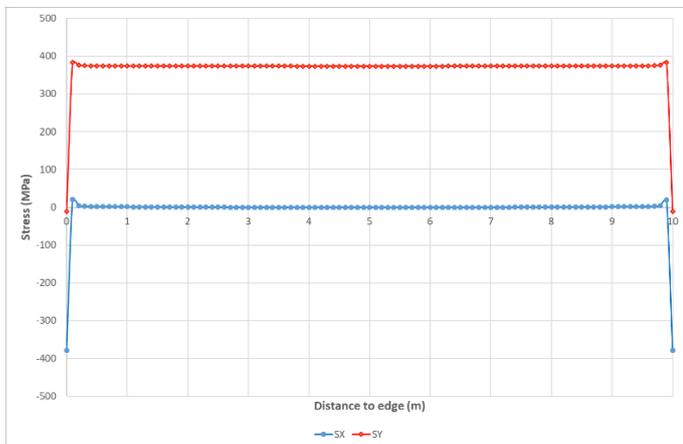


Figure 8 – Longitudinal (S_y) and transversal (S_x) residual stresses along the weld.

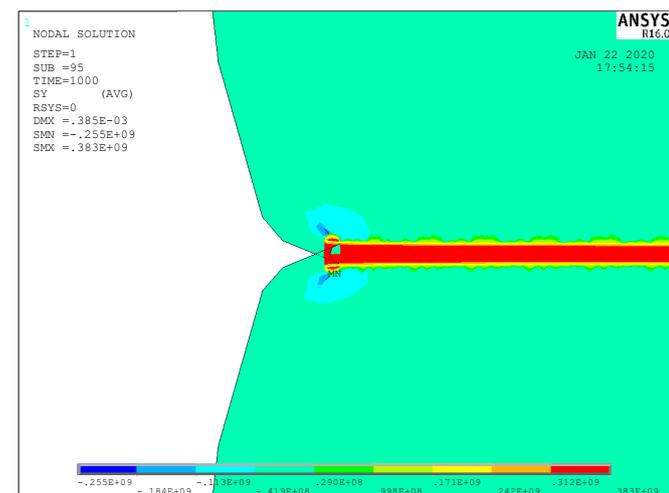


Figure 9 – Detail of longitudinal residual stresses near the top.

3.2.2 Butt weld of plates between stiffeners

Typical plates between stiffeners and frames have different dimensions, been the length of the order of 2.5m and width between 0.5 and 1.0m. In this section it is analysed the case of a plate 2.5 m long and 1.0 m wide with a welding in the middle of the width.

The distribution of longitudinal residual stresses is in accordance with the theoretical model assumed in codes represented by 3 rectangular regions, 1 in tension in the welding at yield stress and compressive stresses to equilibrate in the rest of the plate, as shown Figure 10.

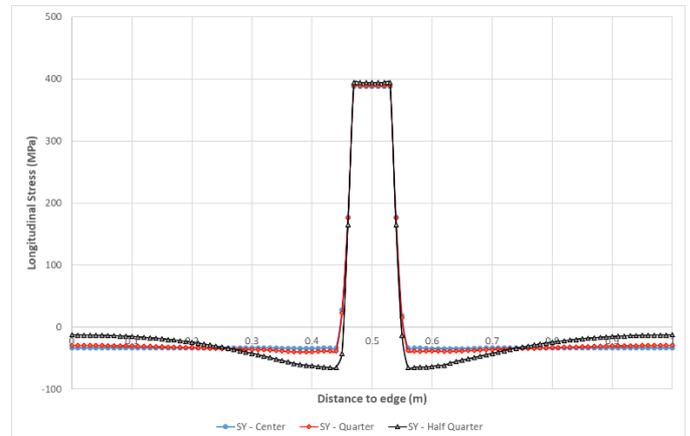


Figure 10 - Distribution of the residual stresses in the middle, quarter and half-quarter of a 10x5m plate with 10 mm thickness and $\eta=1.5$.

The same distribution is presented for a path at a quarter and eight-length showing the appearance of some shear lag effect near the top. The distribution of compressive residual stresses in the middle of the plate is almost constant.

In respect to the top effect on the residual stresses it is found the same response as in previous case. A shorter plate was also used to generate the graphic of residual stresses in the middle of the weld along the length of the plate, Figure 11. It can be observed by the overlap near the top that the length do not affect the length of the perturbed region. Apart from that region with typical dimension of 200mm, longitudinal and transversal stresses remain almost constant.

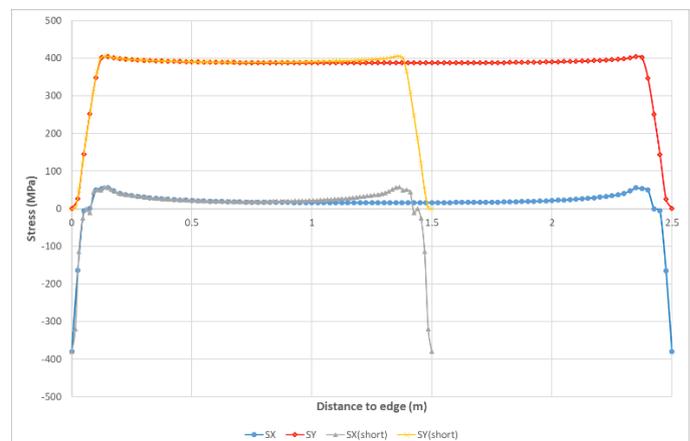


Figure 11 – Longitudinal (S_y) and transversal (S_x) residual stresses along the weld of a 2.5 and 1.5 m long plate.

3.2.3 Thickness relevancy

Variation in thickness for 8, 10 and 20mm produced the results that do not affect the residual stress pattern seen in Figure 12 hence the tensile width is kept con-

stant. In order to guaranty the same width of the tensile zone η parameter changes to 5, 4 and 2, respectively for plate 8, 10 and 20mm thick.

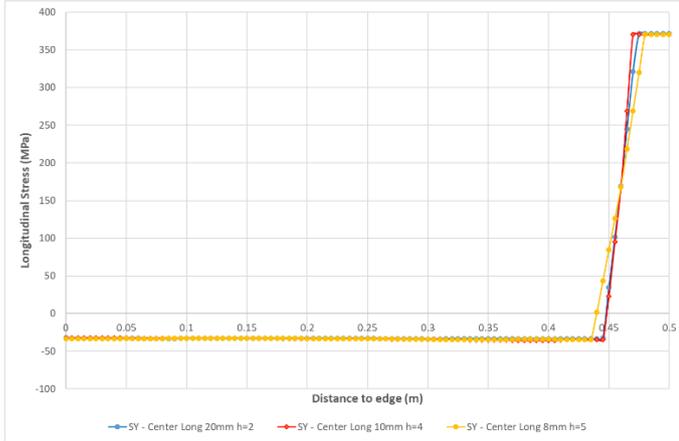


Figure 12 - Comparison between the distributions of longitudinal residual stresses for different thicknesses

3.2.4 Boundary conditions and imperfections

In continuous reinforced panels the boundary conditions (BC) tend to be better represent by constraint condition of the edges due to the rigidity of stiffeners and frames. Applying such conditions the top effect mentioned previously disappears, as may be observed in Figure 13. Also the shear lag due to the proximity of the tops disappears.

The compressive residual stresses are 10% of yield stress.

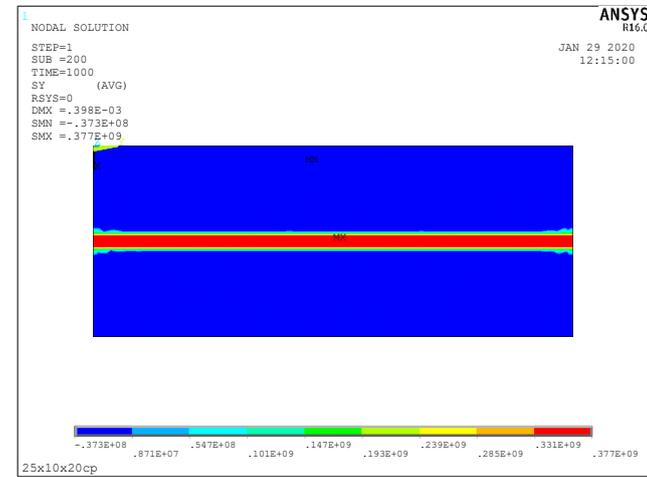


Figure 13 - Distribution of the residual stresses in the longitudinal direction for a plate with 2.5x1.0x0.020 m with $\eta=2$

This model has an initial imperfections amplitude of 4mm before applying the thermal process, introducing displacements on the nodes of the flat plate according to sinusoidal function:

$$\omega = 0.004 \cdot \sin \frac{\pi x}{1.0} \sin \frac{\pi y}{2.5} \quad (4)$$

After the thermal process to generate residual stresses the amplitude was reduced by 0.155mm due to the tensile forces in the middle of the plate on the weld. This reduction is less than 4% of the initial value, but at 500°, before cooling, the variation was

positive by 5% due to compressive stresses in the middle. The shape also changes very much during the whole process.

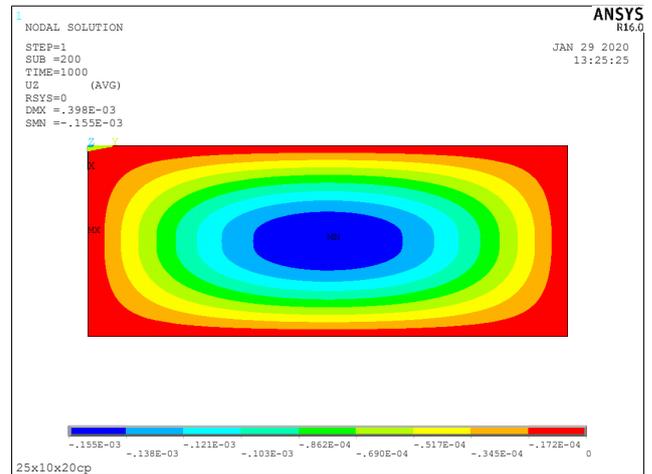
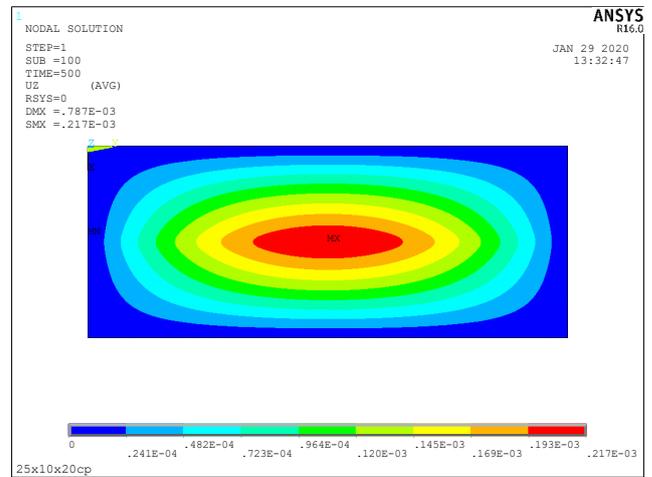
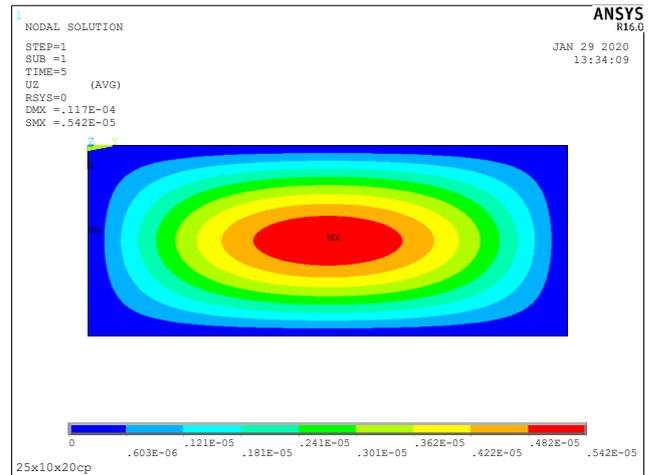


Figure 14 – Variation of out-of-plane deformations for the plate with 4mm amplitude of imperfections; Temperature 25° (top), 500° (middle) and 20° (down).

3.3 Stiffened plates

Stiffened models were also created, with the addition of formerly one and later two stiffeners. The model with two stiffeners was created by mirroring the model with one stiffener along the longer edge, creating a model constituted by a plate with 2 m by 1.6 m and two stiffeners separated by a distance of 800 mm.

It can be noticed that the distribution resembles the theoretical approach. Symmetry is also present, as it would be expected.

Regarding the results for a stiffened model, the resulting distribution of longitudinal residual stresses in the plating is very similar to the ones presented for unstiffened plates with one weld. Figure 15 presents the internal state of stresses before cooling and after it. As can be noticed the induced deformations reverse direction from hot weld to cold weld due to reverse of internal stresses in the toe.

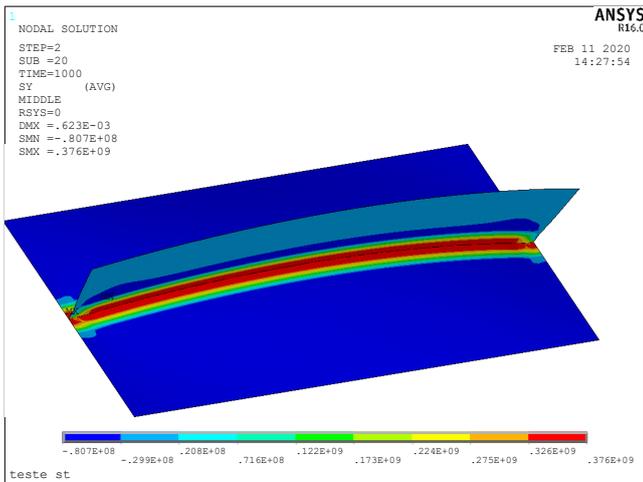
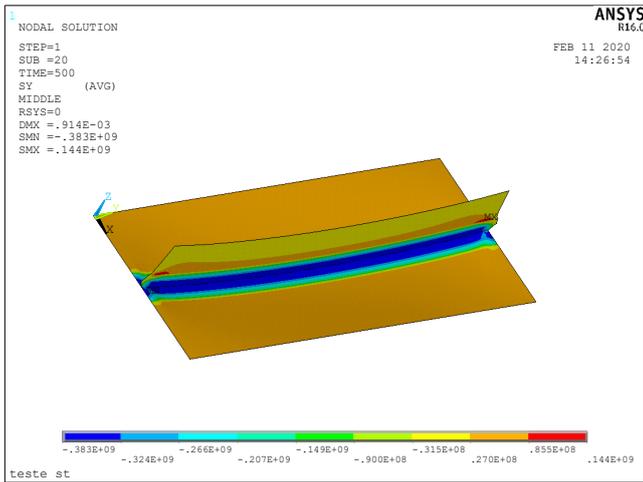


Figure 15 – Internal stresses in longitudinal direction and associated deformations for a perfect flat panel of 2x0.8m with welding strip at 500°C (up) and after cooling (down)

The pattern of residual stresses in the web can be seen in Figure 16. The linear reduction on the compressive stress from the toe is according to theory and is a result of the bending of the stiffened plate.

In Figure 17 can also be seen the distribution across the width of the panel for the model with two stiffeners.

These results are important to acknowledge that the addition of a stiffener to the geometry of the model does not influence the existing stiffener.

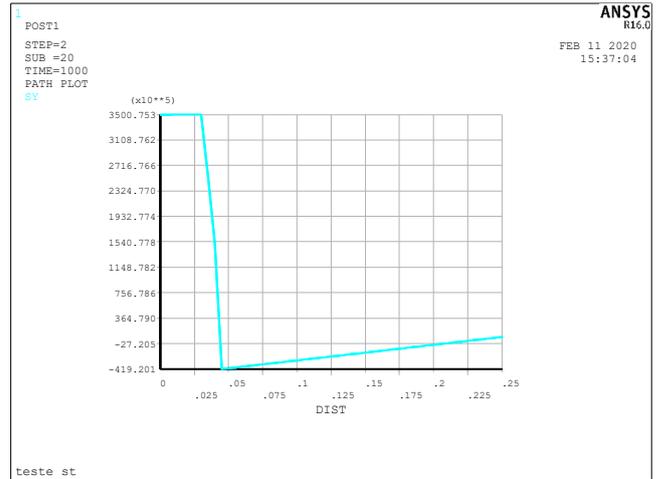
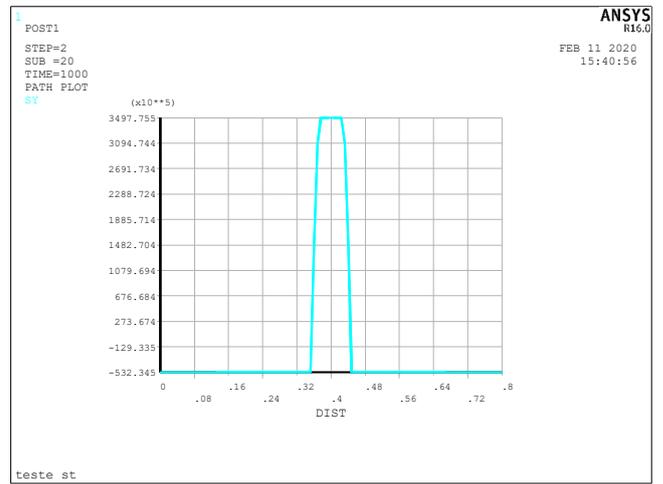


Figure 16 - Distribution of residual stresses across the width of the plate at mid-length (up) and along the web of the stiffener (down)

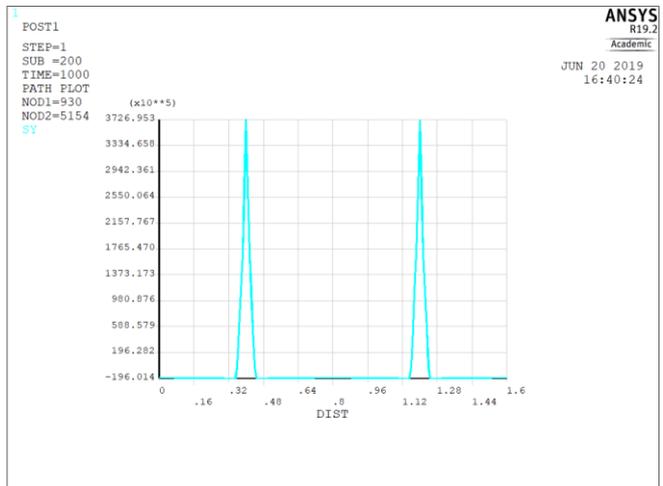


Figure 17 - Distribution of the longitudinal residual stresses on the plate with two stiffeners

4 CONCLUSIONS

Firstly, it can be concluded that the implementation of a FEM model in 2D is capable of producing reasonable results for the distribution of residual stresses. With the model created it is easier to perform simulations faster and have significant results using different geometries at choice. The implementation of

simple geometries and heat inputs allowed for a significant amount of runs to be made, for different thicknesses and modes of initial imperfection.

The successful implementation of a simple yet effective model of inducing residual stresses along several geometries is definitely a major achievement because it allow performed structural analysis in a simplest way.

Regarding the uniform heat input, it was proven to achieve adequate results for the validation in question to be performed.

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