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EFFECT OF INITIAL IMPERFECTIONS ON THE STRENGTH OF RESTRAINED PLATES

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ABSTRACT
Most common studies on the strength of plates under compressive longitudinal loading are related to plates having unrestrained edges which lead to a zero net load in the transverse direction. In ships, the framing system and the continuity of the plating in the transverse direction tend to induce rather different boundary conditions on the 'unloaded' edges, which results in a completely different state of stresses when the external loads are applied. This is due to the surge of a significant level of induced membrane stresses in the direction perpendicular to loading.
In this work, the behavior of long restrained plates under compressive axial loading is analyzed and compared with the one of plates having other boundary conditions. The finite element method is applied for the nonlinear analysis of the plates using a commercial package. 56 cases are considered corresponding to different levels of plate slenderness which ranges between 0.35 and 3.46 covering the practical range of structural plates used in the shipbuilding industry. Various shapes of initial imperfections are considered in order to establish the minimum level of resistance. Also the influence of the magnitude of the distortions associated to each mode is discussed. The study conducted to the establishment of the minimum compressive strength of restrained plates and it defines the expected range of strength's variation due to the magnitude level of distortions.
The biaxial axial state of stresses resulting from these boundary conditions is characterized and its dependence from the plate's slenderness is quantified for the most common type of the hull plating.

INTRODUCTION
The ultimate strength of unstiffened plates under compression depends on a large number of geometric parameters, manufacturing and materials in respect to its mechanical properties. The variation of each parameter in the range of utilization leads, in most cases, to changes of ultimate strength which may reach high values. In a more general way, it may conduct to a deep change in the response curve of the plate, resulting in different stress-strain curves for small changes on the parameters. Such changes require quantification so that they can be included in the design codes [1] and used in structural reliability studies [2].
This study investigates the behavior of typical ship plates with various types of initial imperfections using the finite element method [3]. The influence of initial imperfections on the ultimate strength of simply supported plates with restrained lateral edges is analyzed in respect to the amplitude and mode of distortions. Either the geometry of imperfections or the degree of restraint of the movement of lateral edges may originate quite different response [4], leading to differences on the ultimate strength of the plate that may reach values of the order of 20%. Also the behavior and post collapse resistance are quite conditioned by these two parameters.
The comparison between the results and design codes leads to the conclusion that it is possible to include these parameters in the existing codes by introducing corrective factors enabling accurate prediction of the strength of the plate under compression, once known the initial geometry and the involving structure.
The results obtained are also a source of information for the correct parameterization of reliability models.
The knowledge of the most degrading conditions for the strength of the plate can still create the conditions of inspection that must be met to guarantee the local structural integrity at acceptable level.
Despite the ultimate strength of a plate is sufficient for the design of structural components, the knowledge of their behavior during the loading is very important when trying to

estimate the safety of the structure or to know their overall behavior [5].
Thus, it is imperative to know the average stress-strain curves of the plates when one wants to analyze the strength of more complex structural components such as stiffened plates [6] or thin-walled structures [7].
To meet both needs it was conducted a parametric study using the finite element method, with which it was studied the elasto-plastic behavior of simply supported plates, restrained on the unloaded edges, and subject to longitudinal axial compression.

PLATE ELEMENT MODEL

The structural behaviour of 56 plates was analysed through the finite element method, using ANSYS (2007) software [3].
Each plate has 1200 elements, 8 nodes SHELL93 type and the analysis has elasto-plastic, using Riks method and arc-length convergence.
The plates have a length L of 3000 mm, a breadth b of 1000 mm resulting on an aspect ratio $\alpha(=L/b)$ of 3. The plates are grouped by 9 different thickness t: 100, 60, 50, 40, 30, 25, 20, 15 and 10 mm.
The first four groups have no practical interest on ship structures but their study allows to establish the plate strength on a wide range of plate slenderness, β, which is defined by equation (1), where E is the Young modulus of the material, taken as 200 GPa in this work, and σ_o is yield stress of the material. The normal steel yield stress of 240 MPa was used.

$$\beta = \frac{b}{t}\sqrt{\frac{\sigma_o}{E}} \quad (1)$$

It is assumed that the material has an elastic perfectly plastic behaviour, i.e., without hardening after yielding.
The boundary conditions are simply supported allowing the rotations of all edges, but the linear movement perpendicularly to lateral edges is not allowed. This is a typical boundary condition of the so-called restrained plate [8] by opposition to the constrained plate where the lateral edges are allowed to move in while ensuring that all the points of the edge move the same, remaining straight and without net transverse load applied.
Under these circumstances, the application of an external in-plane force to the tops of the plate generates a biaxial state of stresses due to the Poison's effect and the increase in the out-of-plane deformations of the plate. As a consequence, one has a net global reaction on the lateral edges that changes with the level of the external load applied on the tops.
The initial imperfections have an amplitude w_o of $b/200$, i.e. 5 mm for these plates, and the spatial distribution on the plate is given by the equation (2).

$$w = w_o \sin\frac{m\pi x}{L}\sin\frac{\pi y}{b} \quad (2)$$

The m parameter represents the number of half waves and, in this study, it varies from 1, for the plate with primary or fundamental mode, until 6, corresponding to a wave length equal to the width of the plate since one has α=3.
Figure 1 plots the plate, its mesh and the initial imperfections correspondents to m=6 mode.

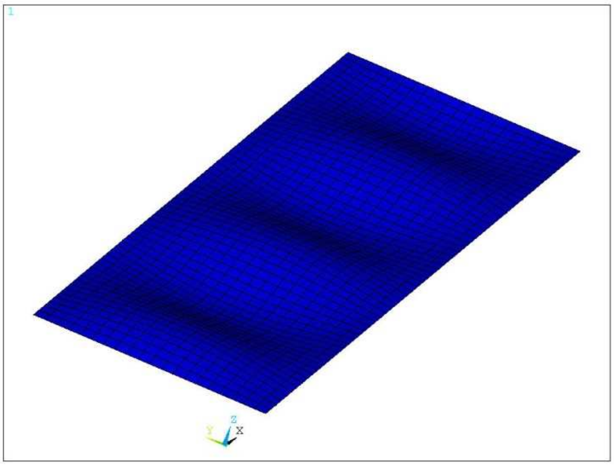

Figure 1– Plate model, mesh and initial imperfections

STRESS STRAIN CURVES OF RESTRAINED PLATES

In the next figures it is presented the relationship between the average stress and the average shortening given by $\Delta L/L$ for the different groups of plates.
The plates of b/t groups equal to 10, 16.7, 20 and 25 are not usual in ship structures, but they are presented here to obtain a better definition of the maximum strength in larger range of plate slenderness and to quantify the degradation of strength due to the increase of the initial imperfections' magnitude in plates with predominant plastic behaviour.
The ultimate strength of plates with b/t=10 and 16.7, Figure 2, is not very dependent from the mode of imperfections m. As expected, due to the restriction on the lateral movement of the edges, the maximum stress achieved is higher than the yield stress and it is very close to the theoretical limit given by the von Mises criteria. This limit is $1.125\sigma_o$ resulting from the equation (3), where ν is the Poisson's ratio.

$$\sigma_x = \frac{\sigma_o}{\sqrt{1+\nu^2-\nu}} \quad (3)$$

Also the initial slope of the curves, usually designated initial structural tangent modulus, is higher than the Young modulus of the material and almost equal to the theoretical value of perfect plates, that is E'=1.099E. The highest structural modulus found was 220 GPa for the plate with b/t=10 and m=1.
However, one should note that plates with b/t equal to 16.7 show already some decrease in the ultimate strength for mode 6.

On plates with *b/t*=20, Figure 3, the strength shows already a marked dependence on the initial imperfection mode. The reduction of the ultimate strength is 9% when passing from a plate with *m*=1 to one with *m*=6 having the same magnitude of initial imperfections. Once the ultimate load is achieved, the load carrying capacity remains almost constant, without any significant load shedding after buckling.

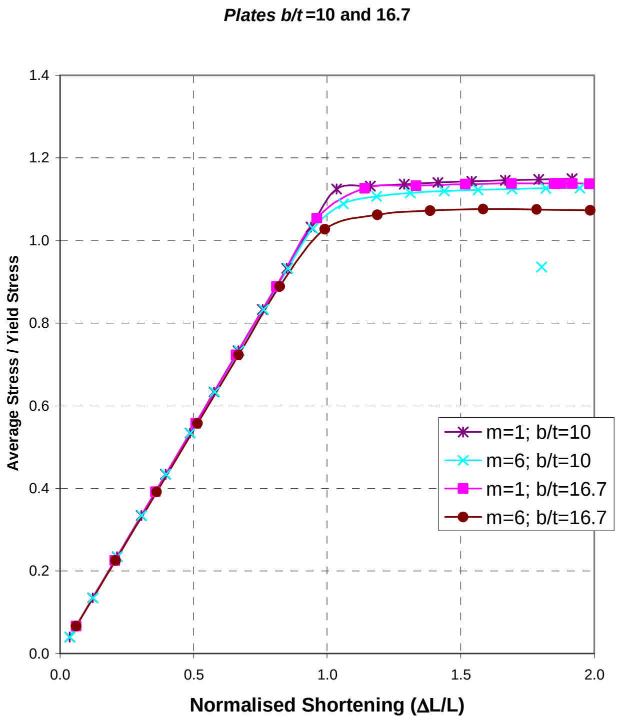

Figure 2 – Plates *b/t*=10 e *b/t*=16.7

The group of plates with *b/t*=25, Figure 4, presents already some load shedding after buckling, specially for the cases of *m* =1 and *m*=2. At high plastic strain the stress-strain curves tend to be independent of the mode of imperfections.

This type of behaviour is still more marked for the group with *b/t*=33, Figure 5. However the load shortening curves tend to be coincident for modes of imperfections $m \geq 4$. For this group of plates, the ultimate stress is already below the yield stress for high modes of imperfections.

It should be noted that this type of plates can already be found on large ships, especially on heavy loaded zones or of great structural importance.

The stress-strain curves keep the same overall characteristics for the group *b/t*=40, Figure 6. The main difference is that the ultimate strength is affected by a reduction factor relatively to the previous group due to the highest slenderness of the plating. The structural tangent modulus is still slightly higher than the Young modulus of steel in the initial stage of loading.

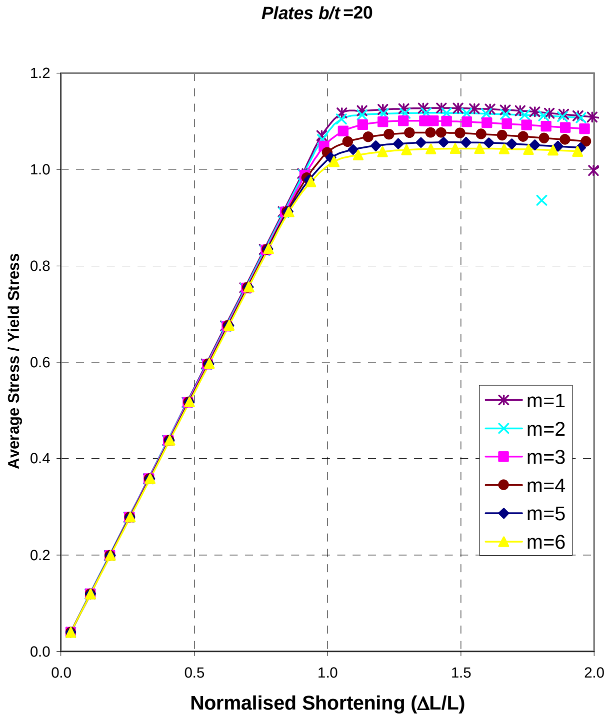

Figure 3 – Plates *b/t*=20

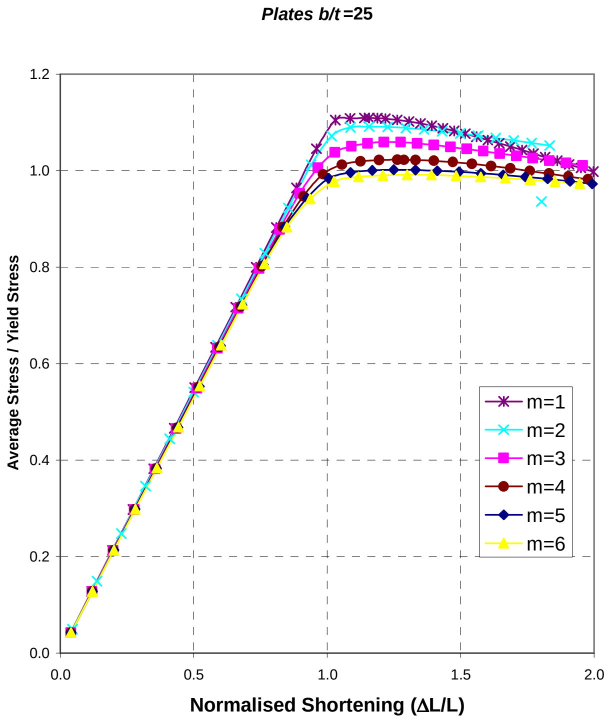

Figure 4 – Plates *b/t*=25

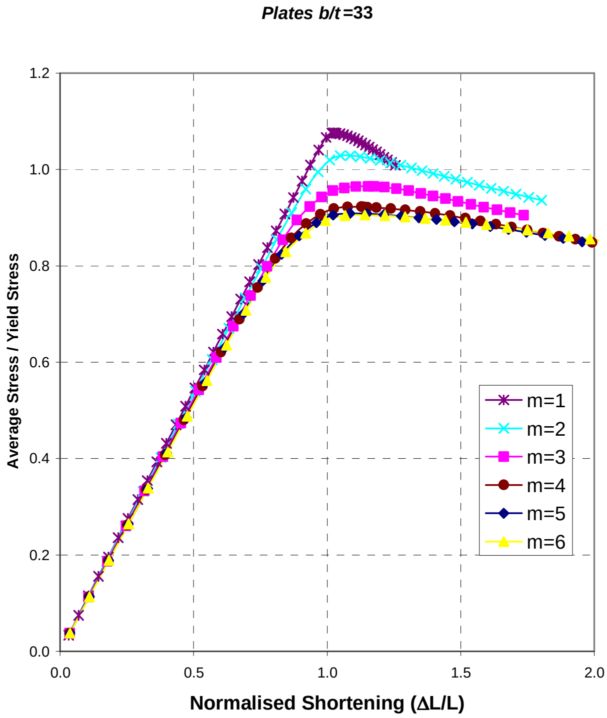

Figure 5 – Plates *b/t*=33

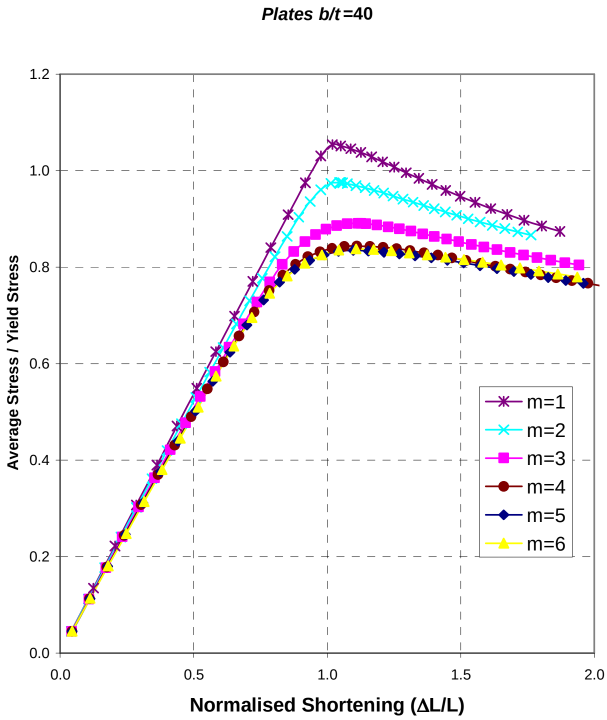

Figure 6 – Plates with *b/t*=40

On more slender plates, *b/t*=50 and above, which are presented in Figure 7 and followings, the elastic instability becomes more important than the plastic one. The plates having the fundamental mode as predominant (*m*=1) collapse with much localized deformations or, alternatively, snap-through to a higher mode of deformations.

The structural modulus becomes rather dependent on the initial mode of imperfections of the plate, decreasing as the mode increases. This dependence becomes more marked as the slenderness increases.

The plates with initial fundamental mode (*m*=1) show a linear behaviour almost until collapse, which occurs now at normalised shortening less than one. The collapse is followed by a great discharge of load, normally associated with a change of mode of deformations. Finally, the plate behaves like the plates with highest modes for shortenings above 1.5.

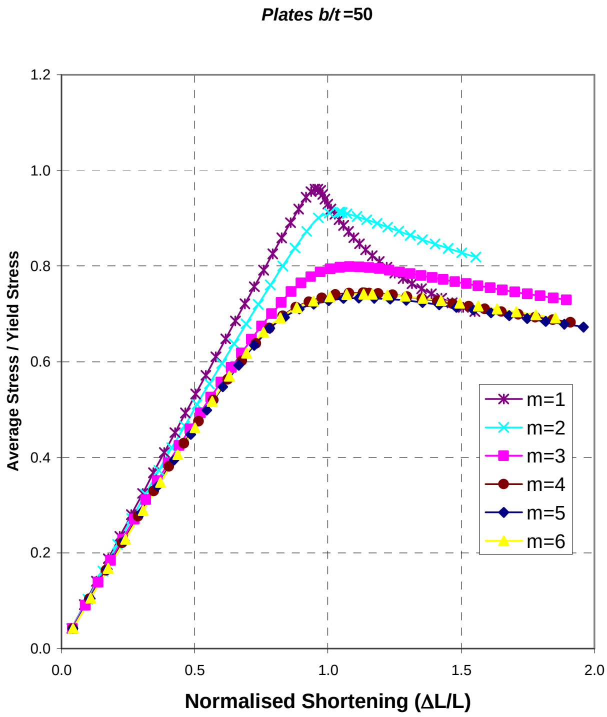

Figure 7 – Plates with *b/t*=50

Increasing the slenderness to *b/t*=67, Figure 8, one has large differences on the ultimate strength between plates with distortions' mode lower than the critical one (*m*=3) and higher than that. These slender plates, which are very common in ships, have a relatively low ultimate strength, slightly above 60% of the yield stress if modes above the critical one are present as initial imperfections.

The nominal shortening tends to occur at values lower than the yield shortening on the cases of plates with *m*=1 and 2.

Very slender plates, presented in Figure 9, have normally a very complex history for the lowest modes of initial imperfections. They tend to snap-through at low normalised shortening and, after that, the behaviour becomes similar to the plate having an initial mode higher than 4. These kind of plates, β=3.46, are not used much in the structural part of the hull because they have very low ultimate strength. However, they may be used on non-important locations of an optimised structure.

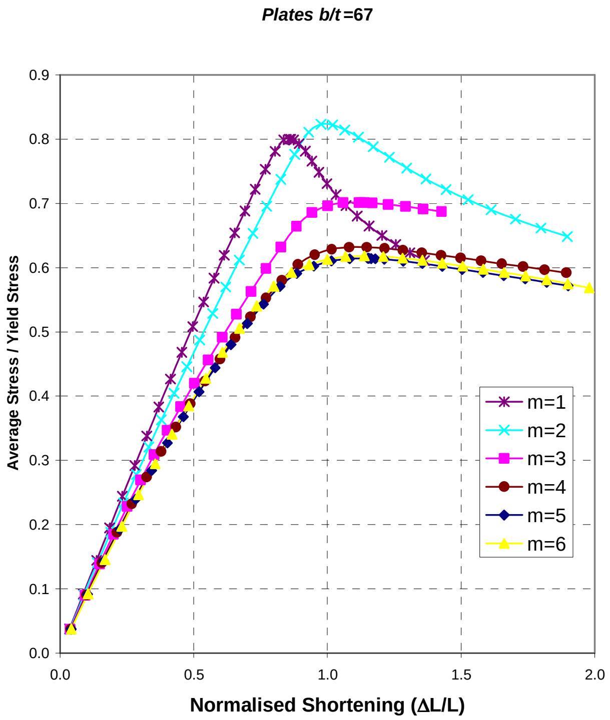

Figure 8 – Plates with *b/t*=67

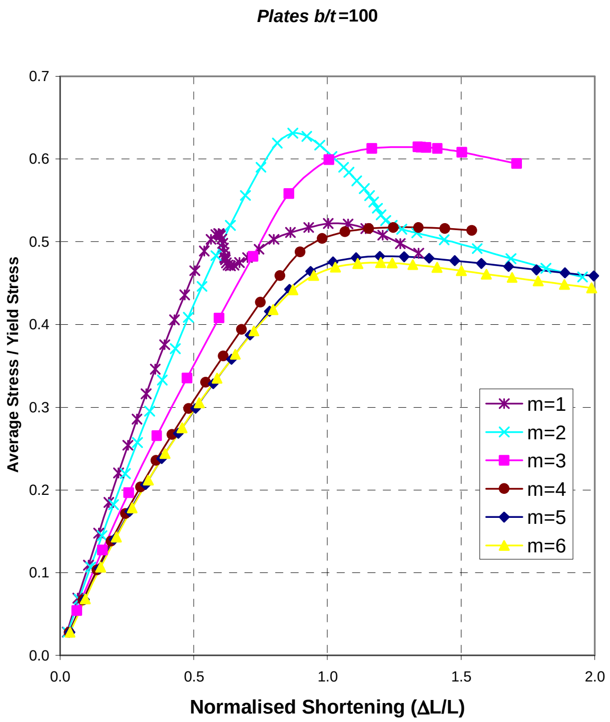

Figure 9 – Plates with *b/t*=100

The maximum average stress is lower than 50% of the yield stress, but the plate with *m*=4 has a significant higher strength than the ones with *m*=5 or *m*=6.
The maximum ultimate stress of this group is higher than 60% of the yield stress, which leads to the conclusion that the difference of the initial mode of imperfections may result in differences of the ultimate strength in the order of 20% for very slender plates.

ULTIMATE STRENGTH OF RESTRAINED PLATES

The results of the ultimate stress normalised by the yield stress of the material are presented in Table 1 for different *b/t* ratios and initial mode of imperfections. In the last two lines it is computed the difference between the maximum and minimum ultimate stress for each thickness (or slenderness), $\Delta\sigma$, and the percentage that it represents relatively to the minimum ultimate stress in each group, as a result of the variation on the mode of initial imperfections.

Table 1 – Ultimate strength of restrained plates

t	**10**	**15**	**20**	**25**	**30**	**40**
b/t	**100**	**67**	**50**	**40**	**33**	**25**
m=1	0,522	0,800	0,961	1,053	1,076	1,109
m=2	0,631	0,823	0,913	0,975	1,028	1,091
m=3	0,615	0,701	0,799	0,891	0,965	1,060
m=4	0,517	0,632	0,744	0,844	0,923	1,023
m=5	0,482	0,614	0,733	0,835	0,909	1,002
m=6	0,475	0,615	0,742	0,838	0,905	0,991
$\Delta\sigma$	0,156	0,209	0,228	0,218	0,171	0,118
$\Delta\sigma/\sigma_m$ (%)	32,8	34,0	31,1	26,1	18,9	11,9

For usual ship's plating, one may conclude that the differences due to changing the dominant mode of imperfections may reach very high values, i.e., as much as 30% variation on strength.
In absolute values, the plates having an intermediate slenderness ($40\leq b/t\leq 67$) are the most sensitive to the mode of initial imperfections. Figure 10 plots these results.
Figure 11 presents the average applied stress versus the normalised shortening relationship for each slenderness and mode of imperfections $m=1+\alpha$.
There are two important remarks to make: first, the great dependence of the curves on the *b/t* ratio (or slenderness) as expected; secondly, the load shed after reaching the ultimate stress is very small and the curves are very flat after collapse. From the structural point of view, this indicates a good level of strength reserve after buckling on more complex structures composed of plate elements.

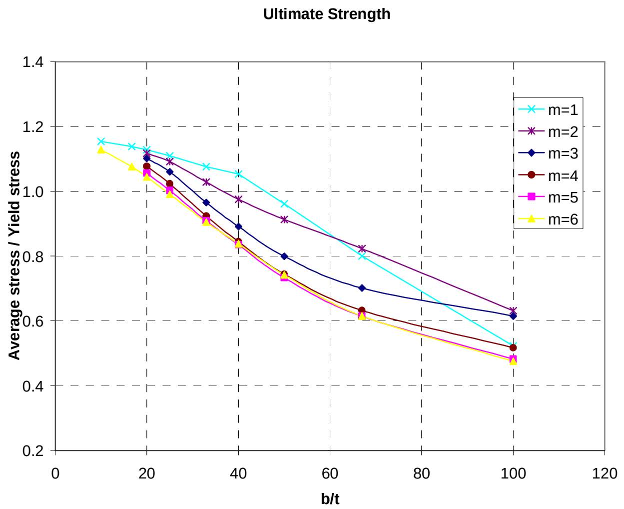

Figure 10 – Ultimate strength of mild steel plates

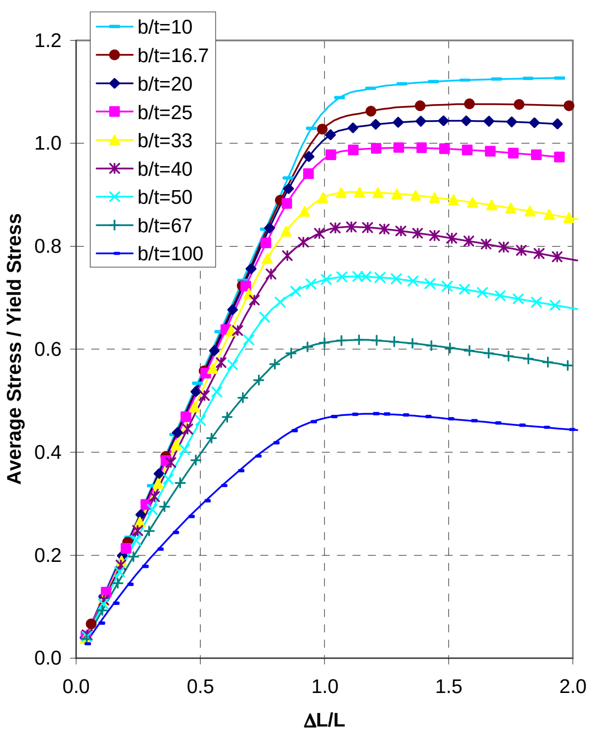

Figure 11 - Curves of minimum stress for restrained plates with amplitude of distortion equal to *b*/200.

In order to compare the ultimate strength of these restrained plates with the ones with simply supported edges free to move, it is present also in Figure 12 the Faulkner's formula [9] for the effective width of unstiffened plates, given by:

$$\phi_f = \frac{2}{\beta} - \frac{1}{\beta^2} \tag{4}$$

It is also compared with the formulas presented by Gordo [5] for the ultimate strength of restrained plates having a critical mode of initial imperfections (m=α) or one order above the critical mode (m=1+α).
The ultimate strength of restrained plates with a critical mode of distortions (m=α) is given by:

$$\phi_{pc} = \beta^{-0.44}; \text{ maximum } 1.07 \tag{5}$$

For a mode above the critical one (m=1+α), the ultimate strength is given by:

$$\phi_{pd} = \beta^{0.02 \cdot \alpha - 0.59}; \text{ maximum } 1.05 \tag{6}$$

These results were obtained with PANFEM finite element analysis software [10] for the study of unstiffened plates and the amplitude of distortions was equal to 0.1tβ. This value is lower than the one used in this study by a factor of 5.

It is obvious from the comparison in Figure 12 that the ultimate strength of simply supported plates is approximately 10% higher than the one for restrained plates with mode of initial imperfection equal to double of the critical mode and amplitude close to the maximum admissible limit, which is an high level of distortions. Only for very stocky plates (β<1) the relation between the two reverts.
Equation (6) is valid to represent the minimum ultimate strength of restrained plates at intermediate slenderness but it is conservative both at very high and very low slenderness values.
It is also obvious that the plate modelling using the critical mode of imperfections is not a warranty of having a minimum strength response and, thus, the final result may become optimistic. This situation is particularly marked at slenderness above 1.5.

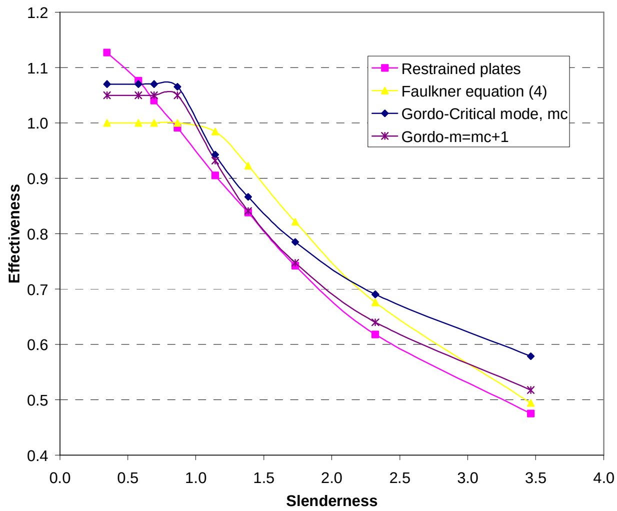

Figure 12 – Comparison with other formulas

AMPLITUDE OF DISTORTIONS AND ULTIMATE STRENGTH

The amplitude of initial imperfections, w_o of equation (2), for each geometric mode contributes largely for the final mode of collapse as well as for the ultimate strength of the plate in the direction of the load. The post-collapse behaviour may also be affected, leading in some cases to a sudden and quick discharge of loading which is normally associated to snap-through of the structure.

It was analysed the behaviour of the most common types of plates of ship structures covering a large range of imperfections for the two most significant modes in terms of strength (m=1 and m=6) and it was quantified the effect on the ultimate strength.

The average load shortening curves are presented in Figure 13 and Figure 16, respectively for stocky plates (b/t=40) and slightly slender ones (b/t=67).

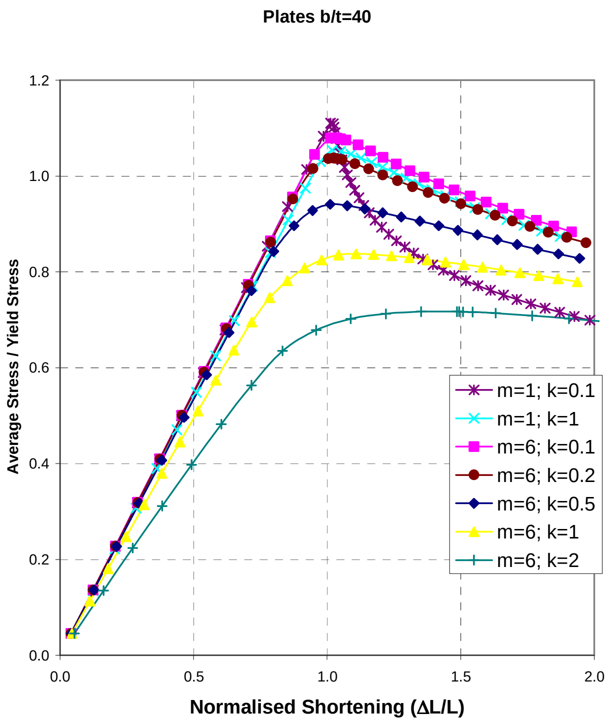

Figure 13 – Effect of amplitude of imperfections on stocky plates, b/t=40.

The amplitude of imperfections, w_p, is represented by the parameter k that multiplies the reference value, b/200, as follows:

$$w_p = k\frac{b}{200} \qquad (7)$$

Stocky plates having the fundamental mode (m=1) of imperfections are not very sensitive to the increase of the amplitude in the pre-collapse range.

The ultimate strength has a small reduction of 6% when the amplitude increases by a factor of 10. However, the almost flat plate (k=0.1) has a deep shed after collapse due to a change in the configuration of deformations. The residual strength of this plate at a normalised shortening of 2 is rather lower than that of the plate with k=1, as can be seen in Figure 13. This difference comes directly from the form of deformation that is developed during the collapse phase.

The plate with small imperfections (k=0.1) develops a deformed shape at collapse characterized by a narrow band where the out of plane deformations are concentrated, as represented in Figure 14, sub-step 14. The deformations before and after collapse are also presented.

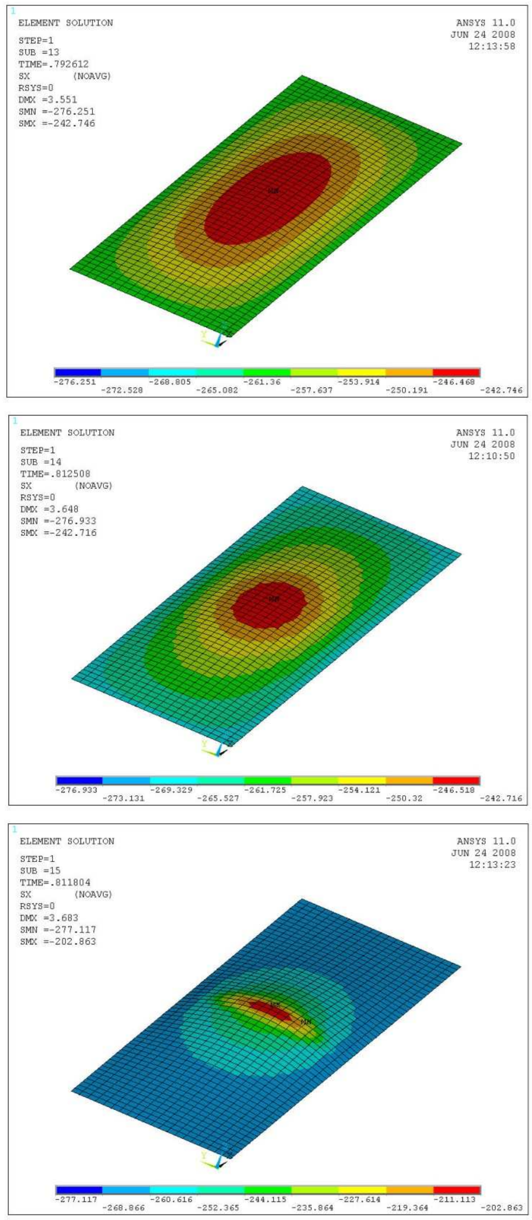

Figure 14 – Evolution of the plate's deformation during collapse, b/t=40 com k=0.1

The plate with high level of imperfections (k=1) tends to develop a complex deformed pattern at collapse, where the amplitude of the mode m=5 becomes dominant together with the fundamental one (m=1). Figure 15 shows the evolution of the out of plane deformations for this case, at collapse, for the steps 14, 15 and 16.

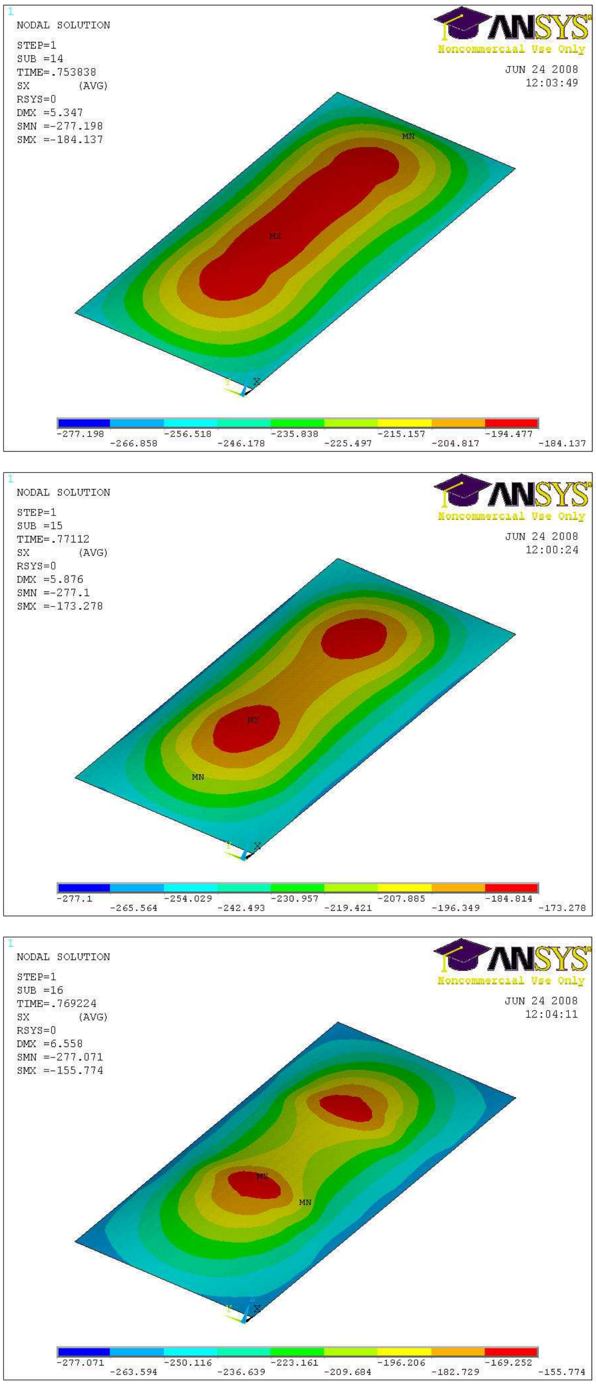

Figure 15 – Evolution of the plate's deformed during collapse, *b*/*t*=40 com *k*=1

However, plates having m=6 keep the pattern through the path load, but show a great decrease of the ultimate strength with increase of the imperfections amplitude. Almost perfect plates (k=0.1) have an ultimate strength of 1.080, but it decreases to 0.838 for k=1, which is a 24% reduction of the strength. A typical damaged plate (k=2) has a further decrease of more than 15%, to 0.717.

For this mode (m=6), the load shed after collapse is smooth and the stress shortening curve tends to be flat for high initial imperfections.

Slender plates present almost the same type of behavior for mode 6 and the reduction of the ultimate strength due to imperfections is of the same level, as shown in Figure 16.

Plates b/t=67, with imperfections in the fundamental mode 1, have a completely different behavior, showing a structural tangent modulus reduction at a shortening of approximately 0.5, which marks the point of elastic instability of the plate. This reduction is higher in the almost perfect plate (k=0.1), resulting in an ultimate strength lower than the one obtained for the plate with m=1 and k=1, when one should expect the opposite for normal cases.

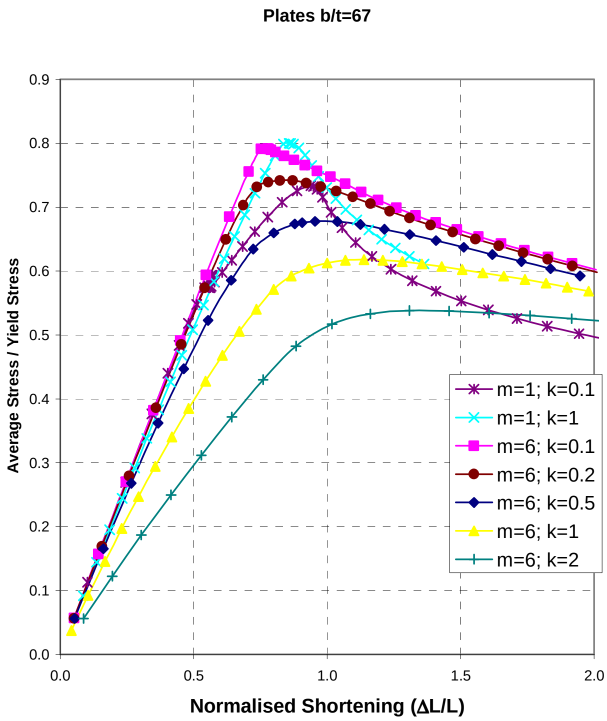

Figure 16 – Effect of amplitude of imperfections on slender plates, *b*/*t*=67.

Figure 17 shows in detail the average stress shortening curve of plate b/t=67 and k=0.1, together with the induced transversal stress due to Poisson effect and the restraint of lateral edges' linear movement. The abrupt change in the slope of the curve is a result of the instantaneous change in the geometry of the imperfections from the fundamental mode to the critical one

(m=α). The snap-through is shown in Figure 18. Initially one has the geometry in the initial elastic range, follow by the change of mode in the first local maximum of the curve.
This local extreme point corresponds to the situation where the middle point of the plate crosses the medium plane. The last plot of the figure shows the geometry after the snap-through at an average normalized stress of 0.58.
One should note the high level of the induced transverse stress at this stage of loading, which reaches 0.13 of the yield stress. It is in fact this high transversal stress that promotes the change of the mode of deformations by buckling. After the bifurcation point, the transversal stress reduces progressively, vanishing close to the point where the ultimate strength of the plate is reached due to the development of large out-of-plane deformations that tends to create transversal tension. In the final phase after collapse, the transversal average stress becomes negative, in tension, reducing the rate of growth of the out-of-plane deformation's amplitude.

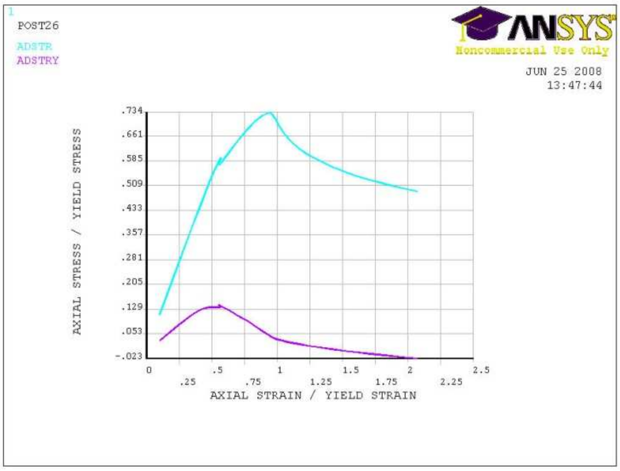

Figure 17 – Average longitudinal stress (ADSTR) of plate b/t=67 with k=0.1 and induced transversal stress (ADSTRY).

INDUCED TRANSVERSAL STRESS

The induced transversal stress due to the restriction of in-plane linear displacement may achieve high levels that affect the behavior of the plates under longitudinal compression. Thus, the state of stresses in the plate is predominantly biaxial.
In Table 2 it is summarized the minimum values of the ultimate stress for each group of plates. Figure 19 plots these results showing that the state of stress is biaxial compressive and the level of the transversal stress may reach more than 30% of the yield stress in stocky plates. The intensity of the induced stress decreases as the slenderness increases and, for very slender plates, the value is residual.
Beyond a slenderness of 3.5, the average induced transversal stress reverts to tensile because the amplitude of deformation reaches high levels that cancel the Poisson effect.

Table 2 – Stress state at collapse on plates with m=6

t	b/t	β	Strain ΔL/L	Ultimate Stress	Transversal Stress
10	100	3.46	1.199	0.475	0.002
15	67	2.31	1.138	0.618	0.096
20	50	1.73	1.134	0.742	0.169
25	40	1.39	1.081	0.838	0.222
30	33	1.15	1.141	0.905	0.259
50	20	0.69	1.479	1.044	0.347

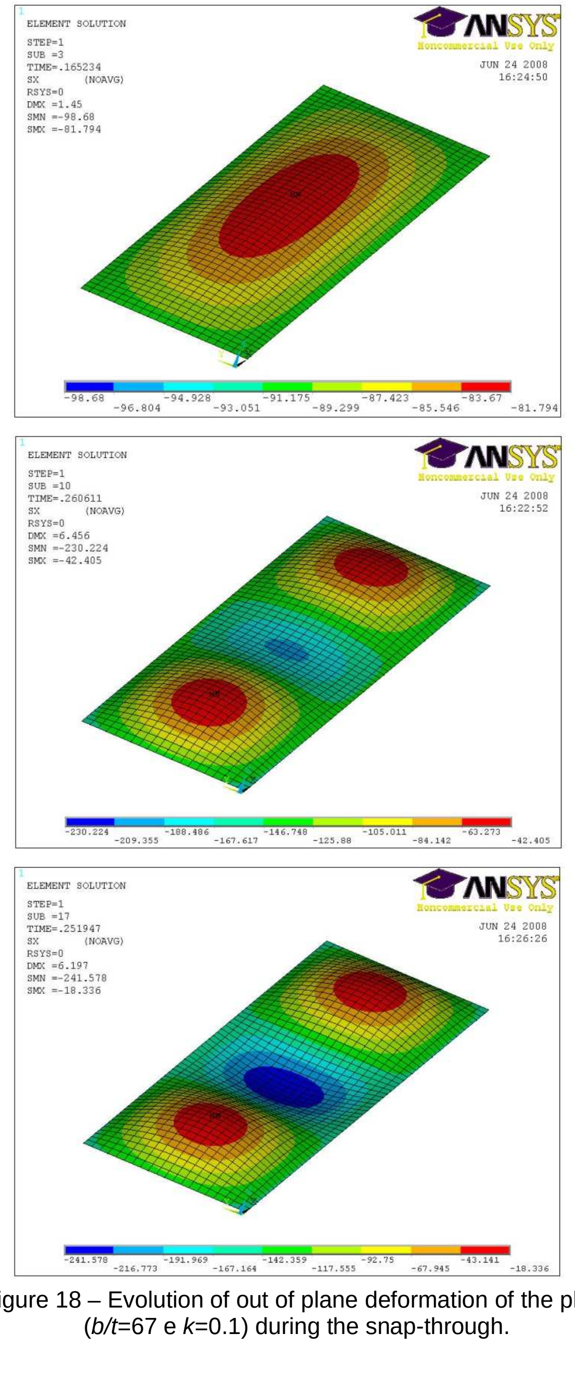

Figure 18 – Evolution of out of plane deformation of the plate (b/t=67 e k=0.1) during the snap-through.

In fact, the ratio between the induced and the applied stress is of the order of magnitude of the Poisson ratio, at initial stage of loading. However, it decreases with increasing shortening until vanishing, and then, the development of large out-of-plane deformations at collapse and beyond generates a tensile state in the transversal direction. Figure 20 shows the evolution of the biaxial state of stress during the loading path on plates with *m*=6 for typical range of ship plating.

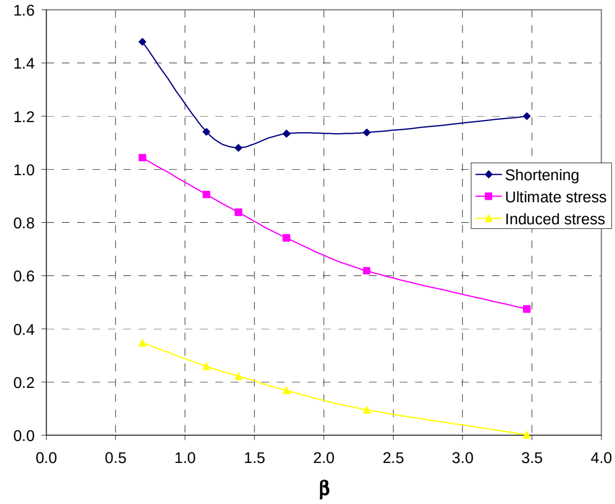

Figure 19 – Ultimate stress and induced transversal stress of plates with *m*=6 and correspondent shortening.

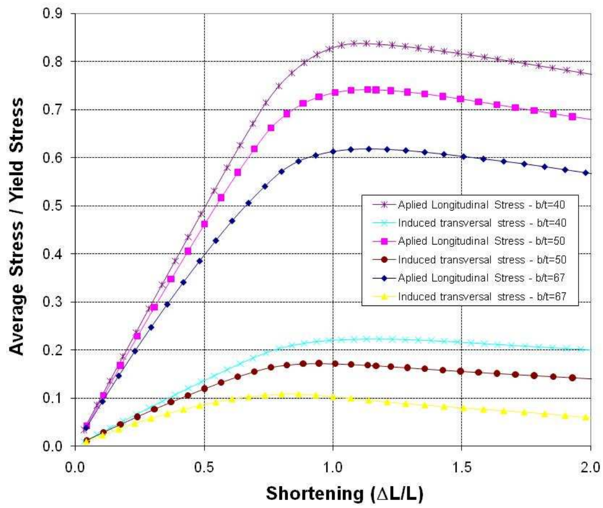

Figure 20 – Longitudinal and induced stress curves for plates with *m*=6.

Transversal stress distribution induced by the longitudinal loading is represented in Figure 21 for the plate b/t=40 at collapse. It is evident that the lateral edges are under compression in a very uniform way. In the central region, the level of transversal stress is very much affected by the influence of the deformed shape of the plate.

In this phase of loading, large plastic deformations are developed, leading to the generation of plastic hinge lines on the locally concave faces of the plate. These hinge lines can be identified easily in Figure 22 and agree with the usual models.
The other plates with similar mode of imperfections but different slenderness behave in a similar way, with a similar pattern of transversal stresses distribution. However, the intensity of the induced stresses decreases as the slenderness increases.

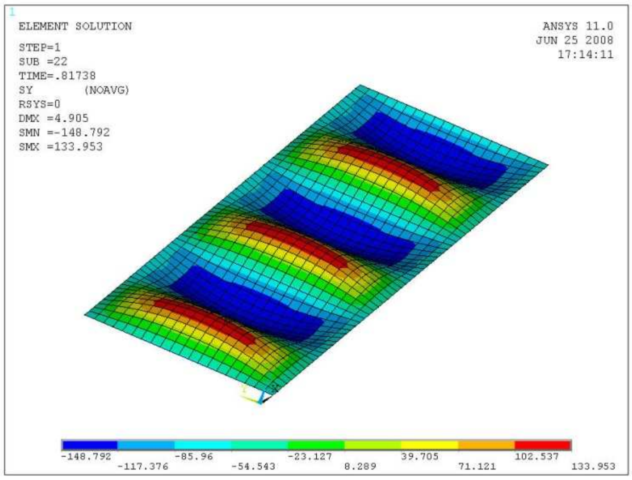

Figure 21 – Transversal stress at collapse in plate with *b/t*=40 e *m*=6

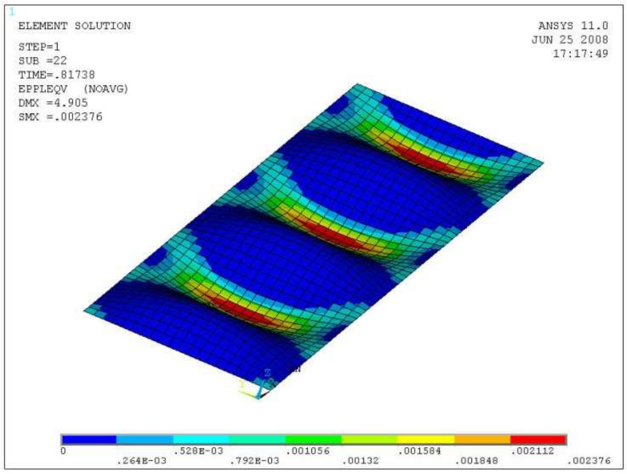

Figure 22 – von Mises plastic strain

CONCLUSIONS

Restrained plates may have an ultimate strength higher than the yield stress for very stocky plates with small amplitude of imperfections.
The ultimate strength depends very much on the predominant mode of initial imperfections. The difference on strength due to changing the mode may reach values of the order of 20% or more.

It was concluded that modes of high order, compared to the critical mode, lead to the minimum of the ultimate strength. The minimum is obtained for plates having a sinusoidal shape with wave length equal to the plate width, $m=2\alpha$.

However, plates with $m \geq \alpha+1$ present a very similar behaviour in the range of slenderness considered in this work. This similarity between the stress shortening curves of plates having predominant high modes of imperfections, higher than 4 for plates of aspect ratio $\alpha=3$, allows to state that local defects or damages with moderate amplitude of defect do not affect significantly the minimum ultimate strength of restrained plates.

The state of stress is predominantly biaxial, compressive. The stresses induced in the direction perpendicular to the loading can reach very high values, generating non negligible forces in the structures of support.

Surprisingly, the minimum ultimate strength of restrained plates is lower than the one of constrained plates, given by the statistical formula of Faulkner, in the usual range of slenderness of ship's plating.

Finally, it was confirmed the validity of the formula for the minimum ultimate strength of restrained plates presented in [5] and given by equation (6) for plates with $m=\alpha+1$.

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