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## Degradation of long plate's ultimate strength due to variation on the shape of initial imperfections

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**ABSTRACT:** The initial imperfections can degrade the strength of structural elements such as plate elements and reinforced plates especially when subjected to compressive loads in the plan. The amplitude of the imperfections is usually considered one of the most important parameter in the quantification of this degradation of the plate's strength. In this study, performed with the finite element method it is shown that the form of the initial imperfections assumed greater importance than its maximum amplitude. Groups are defined with different levels of strength as a function of the shape of initial imperfections and it is quantified the degradation of the longitudinal axial compressive strength due to the increase of the magnitude of initial defects in each of these groups of plate elements both for stocky and very slender. From this study it is concluded that the minimum strength in simply supported plates with restrained edges is obtained when the mode of initial imperfections is greater than the critical mode.

### 1 INTRODUCTION

The plate elements present in marine structures, and particularly in ships, have initial imperfections due to the manufacturing process in steel mills, welding process in steel shops and the efforts to which they are subject in the assembly operations. Studies carried out both on ships and in civil engineering structures quantified the level of distortions in plates trying to correlate it with the geometrical characteristics of the plate.

The presence of distortions in the plate elements causes them to behave differently both in tension and in compression.

In tension, the dominant feature is the variation of the initial tangent modulus. Due to the presence of distortions the initial tangent modulus presents a slightly lower value than the tangent modulus of the perfect plate on similitude of boundary conditions. As a result, the plates in tension belonging to a section of a ship have a reduced rigidity which inevitably decreases the rigidity of the hull. However, for levels of usual distortions, this effect is irrelevant so it can be ignored.

In the particular case of the restrained plates, considered here as the most representative elements of the plate belonging to the continuous panels forming the hull of the ship, the secant modulus is initially lower than the Young's modulus. As the rate of reduction of distortions decreases with increasing elongation, the Poisson effect overlaps the previous one and the secant modulus becomes

greater than the Young's modulus in the elastic domain, as it may be seen in Figure 1.

In compression, the consequences of the distortions are much more pronounced. In fact, their presence on plate elements makes the load-end shortening curves better behaved near the elastic critical load and, in some cases, this load is not clearly identified in the curve. In other words, the sudden collapse featuring almost perfect plates disappears, and load-end shortening curves presents much softer behavior as greater the magnitude of the initial imperfections.

This fact can be verified by the examples of the restrained plates in compression in Figure 1 where the shape and magnitude of initial imperfections was varied as described in Table 1.

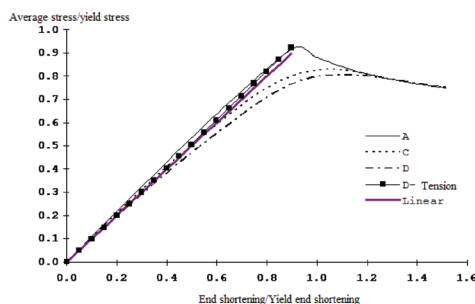


Figure 1. Effect of distortions in simply supported plates with slenderness  $\beta$  of 1.69 and aspect ratio  $\alpha$  of 2.

Table 1. Amplitude of components of plate's initial imperfections with  $\beta = 1.69$  and  $\alpha = 2$ , and maximum amplitude  $w$ .

	$a_{11}$ (mm)	$a_{12}$ (mm)	$a_{21}$ (mm)	$a_{22}$ (mm)	$a_{11}/a_{21}$	$w$ (mm)
A	0.5	0.2	0.2	0.1	2.5	0.682
C	0.1	0.2	1.2	0.1	0.083	1.271
D	0.1	0.2	2.0	0.1	0.050	2.071

Moreover, the shape and amplitude distortion along a plate influence the mode of collapse, causing variations in the final strength of the plate, which are usually negative. Thus it is necessary to quantify the shape and amplitude of distortions characteristics of plates.

### 1.1 Amplitude of distortions

Faulkner (1975) indicated that the maximum amplitude of initial imperfections  $w$  in relation to the thickness  $t$  of the plate,  $\bar{w} = w/t$ , is dependent on the square of slenderness of the plate which is defined by:

$$\beta = \frac{b}{t} \sqrt{\frac{\sigma_o}{E}} \quad (1)$$

where  $b$  is the width of the plate,  $t$  is its thickness,  $\sigma_o$  is the yield stress of the steel and  $E$  is the modulus of elasticity. The values of  $\bar{w}$  vary from  $0.05 \beta^2$  to  $0.15 \beta^2$  and he proposed to use the formula:

$$\bar{w}_F = \frac{w}{t} = 0.12 \beta^2 \left( \frac{t_a}{t} \right) \quad (2)$$

The thickness of the web of the stiffener,  $t_a$ , is always lower or equal to the thickness of the plate  $t$ , and the coefficient of variation of the measurements reduces from 0.6 on stocky plates to 0.3 on slender plates. Very deformed plates may be found in heavily welded panels where the normalized amplitude of distortions may reach  $0.4 \beta^2$ .

Guedes Soares (1988) suggested a mean value of Faulkner's results for warships and of Antoniou (1980) for merchant ships to be used in design stage,  $0.11 \beta^2$ . In fact, Antoniou et al. (1984) proposed a linear dependency of slenderness that could include other less important parameters like ratio between thickness of plate and stiffener, aspect ratio and welding characteristics:

$$\bar{w} = \frac{w}{t} = 0.238 \beta - 0.177 \quad (3)$$

Latorre & Birman (1986) noted that this formula based on the statistical analysis of the data

base predicted very low values when compared to others databases from different sources.

Kmiecik et al. (1995) also found a linear dependency between the maximum amplitude of the imperfections ( $w_{om}$ ) and the slenderness given by:

$$\frac{w_{om}}{t} = 0.0083 \frac{b}{t} - 0.1989 \quad (4)$$

Its database does not allow to find any dependency between the amplitude of each mode of imperfections and the slenderness because of the reduced numbers of points available for each plate. Nevertheless, a good correlation was found for the amplitude of the critical mode and the aspect ratio of the plate:

$$\frac{w_{oc}}{t} = 0.1336 - 0.0308 \frac{a}{b} \quad (5)$$

Bonello et al. (1991) proposed another model based on the probabilistic nature of imperfections for ship made of mild steel. The method assume that the amplitude of distortions normalized by the thickness of the plate has two contributions: a deterministic one which is the mean value of distortions given by eq. (2) considering equal thickness for plate and stiffener, and the second one is a normal distribution function having a standard deviation of:

$$s = \bar{w}_F \cdot (0.675 - 0.118 \beta) \quad (6)$$

Thus, the distribution of imperfections in terms of magnitude is given by:

$$\bar{w}_B = \bar{w}_F + N(s) \quad (7)$$

where  $N(s)$  represents the normal distribution of random variable  $\bar{w}_B$  with a mean value  $\bar{w}_F$  and standard deviation  $s$ .

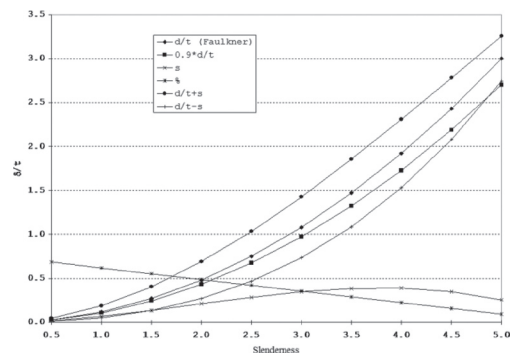


Figure 2. Comparison of different formulations for amplitude of distortions.

From Japan there are also some works related to the quantification of magnitude of imperfections and of their modes of deformations (Ueda & Yao 1985; Ueda et al. 1977).

## 2 EFFECT OF DISTORTIONS

The effect of distortions on the ultimate strength of unstiffened plates have been the focus of research along time.

Carlsen & Czujko (1978) pointed out that imperfections may have a positive or negative effect on strength depending on their magnitude and shape, or the dominant mode. However the increase on the strength is very much affected by any local deformations, thus it should not be incorporated in design (Mansour et al. 1990).

According to Murray (1945) and Dwight (1972) the degrading effect of small distortions in long plates,  $\bar{w} < 0.23$  and  $\omega \gg 4$ , should not be considered, while Frieze et al. (1977), Dwight & Ractliffe (1969) concluded that the effect has to be considered for larger magnitude,  $\bar{w} > 0.3$ .

Guedes Soares (1988) quantified statistically the loss of strength of the imperfect plate due to initial imperfections and residual stresses. He concluded that the ultimate strength of the 'perfect' plate with restrained edges depends on the slenderness by:

$$\phi_{ws} = \begin{cases} \frac{2.16}{\beta} - \frac{1.08}{\beta^2} & \text{for } \beta_o \geq 1 \\ 1.08 & \text{for } \beta_o \leq 1 \end{cases} \quad (8)$$

This formula gives values above unit for very stocky plates which is result of the plane state of stress when the edges are obliged to remain straight. The correction on the ultimate strength due to imperfections was computed as:

$$\phi_d = 1 - (0.626 - 0.121\beta) \bar{w} \quad (9)$$

Thus the ultimate strength of the imperfect plate is:

$$\phi_{us} = \phi_{ws} \cdot \phi_d \quad (10)$$

### 2.1 Model and boundary conditions

This study on the influence of distortions on the ultimate strength of simply supported restrained plates is based in finite element method using PANFEM software (Jazukiewicz et al. 1990).

There are three possibilities for the edges support of simply supported unstiffened plates: unrestrained, constrained and restrained. The former allows the in-plane movement of which point of

the unloaded edges freely; in constrained plates, the unloaded edges are free to move in-plane but all points have the same displacement, thus the total force in the edges are null; restrained planes have zero in-plane displacement, so the induced force in the lateral edges may be different than zero and large membrane stress in the transversal direction may developed during the longitudinal loading.

It was considered different aspect ratios of the plating and several modes of imperfections corresponding to the development of the shape of imperfections in Fourier series, given by eq. (11) and represented in Figure 3.

$$w(x, y) = \sum \sum a_{ij} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) \quad (11)$$

### 2.2 Intermediate to stocky plates

Table 2 presents the values of these components of imperfections for a plate with aspect ratio of 2 and a slenderness of 1.69.

Models A and B emphasize the fundamental component corresponding to an elastic buckling stress of higher order, *primary* group, while C and D models feature the maxima amplitude in the second longitudinal component corresponding a collapse mode leading to the minimum critical stress, *critical* group. The amplitude ( $w$ ) of imperfections increases from A to D. So, it is possible to analyze the influence of amplitude and the mode of imperfections. It also presents the prediction of the ultimate strength for this plate by the approximate method of Gordo and Guedes Soares (1993) that considers average distortions of  $0.11\beta_o^2$ .

On the right side of the same table the results for the ultimate strength of the plate,  $\phi$ , are presented together with its correspondent normalised shortening,  $\epsilon_u$ , the normalised shortening leading to the initiation of plasticity,  $\epsilon_{pl}$ , and mode of collapse pointing out the eventual transition during loading.

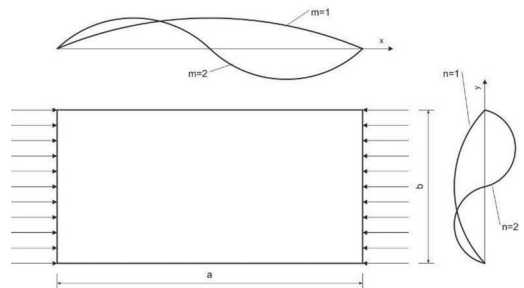


Figure 3. Plate model and shape of imperfections.

Table 2. Initial imperfections on a plate of 5 mm thick, a length of 500 mm and 250 mm of width. Longitudinal strength of rectangular plates with different levels of imperfections. \*Value related to the loss of effectiveness of the plate and not the initiation of plasticity.

$\beta = 1.69$ $\alpha = 2$	$a_{11}$ (mm)	$a_{12}$ (mm)	$a_{21}$ (mm)	$a_{22}$ (mm)	$a_{31}$ (mm)	$a_{11}/a_{21}$	$w$ (mm)	$w/t$	$\phi$	$\bar{E}_u$	$\bar{E}_{pl}$	Mode
A.M.	—	—	—	—	—	—	1.57	0.314	0.833	1	0.35*	$m = 2$
A	0.5	0.2	0.2	0.1	—	2.5	0.682	0.1364	0.886	0.888	0.85	$m = 1 \rightarrow 2$
B	1	0.2	0.2	0.1	—	5	1.066	0.2131	0.917	0.934	0.84	$m = 1 \rightarrow 2$
C	0.1	0.2	1.2	0.1	—	0.08	1.271	0.2541	0.798	1.1	0.78	$m = 2$
D	0.1	0.2	2	0.1	—	0.05	2.071	0.4141	0.766	1.199	0.84	$m = 2$
E	1	0.4	0.4	0.2	—	2.5	1.364	0.2729	0.886	0.917	0.82	$m = 1 \rightarrow 2$
F	0.1	0.2	0.6	0.2	—	0.17	0.716	0.1431	0.829	1.032	0.75	$m = 2$
G	0.8	0.4	0.8	0.2	—	1.0	1.449	0.2897	0.814	0.955	—	$m = 2$
Q	—	—	0.01	—	0.5	—	0.5	0.1000	0.816	0.933	0.74	$m = 3$
R	—	—	0.05	—	1.0	—	1.0	0.2000	0.761	1.096	0.66	$m = 3$
S	—	—	0.1	—	2.0	—	2.0	0.4000	0.702	1.219	0.72	$m = 3$

The plates of the primary group (A and B) have a higher maximum strength than the critical group (C and D), as would be expected from relative amplitudes of the various Fourier components. The variation within each group is also different.

In the primary group the increase in the amplitude of imperfections leads to an increase in resistance due to the relative increase in the prevalence of the 1st order mode ( $m = 1$ ). The plate A has a ratio  $a_{11}/a_{21}$  of 2.5 while on plate B this ratio is 5. Note that maintaining the ratio of the amplitudes of components and increasing the maximum amplitude variation do not change much the ultimate strength as shown by plates A and E.

At the critical group it is otherwise. The increase on the amplitude of imperfections decreases the stiffness and the ultimate plate strength, as shown in Figure 4. This conclusion is consistent with the normal effect of distortion on the ultimate strength.

For this slenderness,  $\beta = 1.69$ , an increase of 63% on the magnitude of distortions there is a degradation of strength of 4%, as given by plates C and D. It has to be noted that Faulkner's formula, eq. (14), predicts an ultimate strength of 0.833 and the linear regression of the critical group's results gives the following expression:

$$\phi_u(1.69) = 0.860 - 0.230 w/t \quad (12)$$

Introducing the Faulkner's prediction and highlighting the degradation due to initial imperfections, one has:

$$\phi_u = 1.032 \phi_F \left( 1 - 0.267 \frac{w}{t} \right) \quad (13)$$

The 'perfect' plate ( $w = 0$ ) is 3.2% larger than Faulkner's formula and 4.6% smaller than Guedes

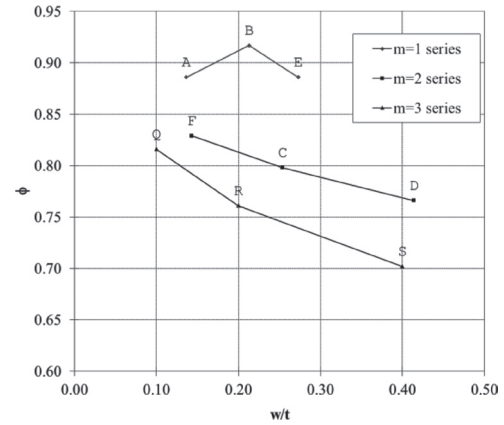


Figure 4. Ultimate strength versus initial imperfections of plates with  $\beta = 1.69$  e  $\alpha = 2$  for different shape of imperfections.

Soares equation for 'perfect plate', eq. (8). This result is not surprising since Guedes Soares used a database that includes all modes of imperfections and the inclusion of primary group results increases the average ultimate strength's prediction.

$$\phi_F = \frac{2}{\beta} - \frac{1}{\beta^2} \quad (14)$$

Figure 5 present the average stress end-shortening curves of the plates under consideration and their comparison with the approximate method based on the effective width (Gordo & Guedes Soares 1993).

The plates where the primary mode is dominant have an increase stiffness and structural modulus

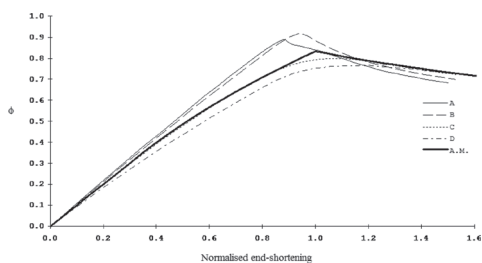


Figure 5. Normalised average stress average end-shortening of plates with  $\alpha = 2$  and  $\beta = 1.69$  having primary and critical mode of imperfections and comparison with Approximate Method (AM).

both in the pre collapse and collapse domain. Only after the collapse these plates have less resistance and more negative tangent modulus than the plates with initial imperfections having a critical mode of imperfections, in this case  $m = 2$ .

However, one should note that this higher slope of the structural tangent modulus after collapse is only apparent, since the collapse of these plates is fundamentally local. Thus the end-shortening in the edges is variable along the plate edge, while at the critical group exists much greater uniformity of that shortening. Thus the local shortening in the region collapse is certainly greater than indicated in the figure.

The approximate method represents quite well the behaviour of plates with initial imperfections averages  $m = \alpha$  in the areas of pre and post collapse. In the collapse zone the gap is slightly larger though not exceed more than 5% error. The value found for the ultimate shortening by the approximate method ( $\epsilon_u$ ) is, curiously, an intermediate value between the ultimate shortening of the plates of the two groups, less than  $0.95\epsilon_0$  on the plates of the primary group and higher the  $1.1\epsilon_0$  to the plates of the critical group. This confirms the suitability of the method if one takes into account that the curves of the critical group are virtually horizontal in this area, that is, have a structural tangent modulus virtually nil. One can thus conclude that the initial hypothesis of the method is valid to assume that the concept of effective width (equation (14)) is applicable in the post collapse phase, simply using the effective slenderness associated with real average shortening instead of using the nominal plate slenderness.

The marked differences in behaviour and strength due to the amplitude and shape of the distortions of these two groups raise the question whether there are two distinct groups of resistance associated with them, or, on the contrary, there is a continuity of the ultimate strength when passing from one to the other.

In this sense it was studied a plate with equal amplitude of the first and second longitudinal components, plate G, and with amplitude of the imperfections of the same order of magnitude of the plates E and C. The plate ultimate strength is similar to the one of critical plate C, +2.3%, and well below one of the plate with a dominant primary mode, -8.1%.

The collapse in plate G develops locally in the area where initial imperfections are smaller. This behaviour is common to all plates in which the failure is associated with a change in the deformed shape and is somewhat difficult to explain, since the areas where one has the greatest initial imperfections are subject to higher stresses at the edges due to a worse transverse load distribution in the pre collapse phase. Furthermore, in the central part of the area with the largest amplitude distortions the applied bending moment is apparently increased due to the larger arm; however, the force per unit of length applied is smaller than in the central part of the area with less distortion. The final plate deformation seems to point therefore to greater importance of uniformity of stresses instead of greater local amplitude of the imperfections. This uniformity of stresses causes that the “critical” strain is hit of at lower shortening and consequently the local buckling phenomena are felt more strongly in these areas, while in the zone of greater initial imperfections plasticity of the edges is the dominant phenomenon even in the case of intermediate plates.

Finally consideration is given to the behaviour of plates having a ternary initial mode, i.e., a higher dominant mode than the critical one,  $m = \alpha + 1$ . Thus, three plates were analysed (Q, R and S) with dominant mode equal to three and different amplitudes of imperfections.

It was concluded that the failure mode coincides with the initial mode and the resistance of these plates is lower than the strength of plates with a critical failure mode, confirming the results obtained for restrained plates having an aspect ratio of 3 (Gordo 2015). It was also possible to identify a reduction in resistance with increasing amplitude imperfections. The symmetry breaking was assured by the introduction of a small component in critical mode.

Using the technique already used to establish the equation (13), the maximum resistance for this failure mode ( $m = 3$  with  $\alpha = 2$ ) can be approximated by:

$$\phi_u = 1.015\phi_F \left( 1 - 0.435 \frac{w}{t} \right) \quad (15)$$

This expression indicates a lower resistance of the ‘perfect’ plate associated with this failure mode



as can be seen in Figure 4 and a further reduction of strength due to distortion compared with plates with a critical failure mode which is present by equation (13).

### 2.3 Very slender plates

The strength of the slender plates with a width to thickness ratio ( $b/t$ ) of 100 has interesting characteristics for the failure mode and its dependency on initial imperfections.

In plates A, C and E belonging to the critical group with mode of initial imperfections in way of the critical failure mode, the increase of initial imperfections reduces the ultimate strength, as is apparent from the results shown in Table 3. However this reduction is very low, 1.5%, between the extremes of different levels of distortions considered.

In contrast, the plates of the primary group, B, D and F, have a resistance increase with increasing distortions; the ultimate strength variations are very strong in this group due to change in the failure mode with increasing distortions, as shown in Figure 6.

The plate with smaller distortion D has a relative maximum in mode 1 but immediately degenerates to mode 3 in order to reach maximum strength (0.518) in this mode. One may note perfectly into the sequence shown in Figure 7 that the first maximum occurs at the beginning of the development of the 3rd mode and re-established the ascending

load phase when the deformed half transposes the base plate plan, acquiring an almost perfect ternary mode in final collapse.

The average distortion plate (B) reaches the collapse mode 1 (0.590, i.e. 13% more than the former) after assuming a collapsed configuration that is a combination of the second and fourth modes  $2 + 2 \times 4$ , or in other words, an imperfect ternary degenerate mode of a half wave is larger than the other two as shown in Figure 8.

On the plate with large imperfections, F, dominates the primary mode throughout the range of extensions despite the emergence of a ternary component, but, however, it is not developed sufficiently to become dominant. As a result of this the ultimate strength of plate F is 20% higher than the plate B, which is a variation of 37% between the two extremes of level of distortions discussed.

Faulkner's formula predicts a strength of 0.504 for this slenderness which is very close to the values found for plates with initial tertiary mode. The strength variation on this group with the level of initial imperfections may be expressed by  $\phi_u(3.38) = 0.517 - 0.024w/t$ , that can be transformed into:

$$\phi_u = 1.025\phi_F \left( 1 - 0.046 \frac{w}{t} \right) \quad (16)$$

This strength reduction factor ( $-0.046$ ) is the biggest one between the three groups considered but, despite that, it is small in absolute value when

Table 3. Initial imperfections on plates of 2.5 mm thick, a length of 500 mm and 250 mm of width and their longitudinal strength and mode of collapse.

$\alpha = 2$	$a_{11}$ (mm)	$a_{21}$ (mm)	$a_{31}$ (mm)	Maximum (mm)	$w/t$	$\phi$	$\bar{\epsilon}_u$	Mode of collapse
A	0.5	3		3.354	1.341	0.59	1.152	m = 2
B	3	0.5		3.148	1.259	0.634	0.792	m = 1→2+4
C	0.25	1.5		1.678	0.671	0.593	0.999	m = 2
D	1.5	0.25		1.574	0.63	0.518	1.036	m = 1→3
E	1	6		6.708	2.683	0.585	1.367	m = 2
F	6	1		6.295	2.518	0.711	0.863	m = 1
G	–	0.025		0.025	0.01	0.632	1.096	m = 2
H	0.025	–		0.025	0.01	0.527	1.063	m = 3
M	0.025	0.002		0.025	0.01	0.6	0.896	m = 2
O	1.9	1.9		3.291	1.316	0.562	0.979	m = 2
P	2.4	1.2		3.118	1.247	0.547	0.926	m = 2
Q	3	0.75		3.273	1.309	0.617	0.79	m = 1→2+4
R	2.7	0.9		3.118	1.247	0.536	0.866	m = 2
I		0.5	3	3.432	1.373	0.485	1.543	m = 3
J		3	0.5	3.353	1.341	0.564	1.112	m = 2
K		–	0.025	0.025	0.01	0.589	0.872	m = 2
L		0.13	0.75	0.858	0.343	0.51	1.152	m = 3
N		0.25	1.5	1.716	0.687	0.498	1.316	m = 3

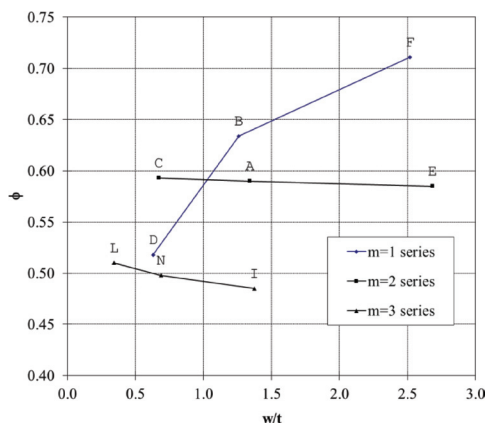


Figure 6. Dependence of ultimate strength on initial imperfections for plates with  $\beta = 3.38$  e  $\alpha = 2$  by groups of shape of imperfections.

compared to the variation of strength associated with different predominant modes of imperfections. Nevertheless, the reduction factor is 6 times bigger on the stocky plates considered in the previous section, eq. (13).

Equation (7) provides greater importance of distortions in both cases with an expected reduction of the double and quadruple in comparison to the reduction estimated in this section to semi thick and slender plates, respectively.

The analysis of the almost perfect plates was performed using an initial imperfection 1% of the thickness on the primary modes, plate H, and on the critical mode, plate G. The latter plate has the same mode of deformation during the entire loading path, however, it presents a major structural change in the tangent modulus at intermediate loading level, as shown on the right side of the Figure 10 representing the stress-strain curve.

Plate H has a ternary failure mode. The initial imperfections has a primary shape and the change of mode to ternary mode occurs on elastic domain due to the existence of transverse tensile stress on the middle of the plate as result of increasing deformation. The transition to the critical mode ( $m = 2$ ) is unable due to the total symmetry of the model.

To test this last statement it was generated a plate similar to H but with a small disturbance that breaks the symmetry, plate M. The shape of the plate during compressive loading evolved from the dominant primary mode in the initial phase, through a premature ternary mode to which corresponds to a relative maximum of the stress-strain curve with the value of 0.365 which is very close to the elastic critical stress for  $m = 3$ , ( $\phi_{c3} = 0.370$ ), having degenerate to the critical mode,  $m = 2$ , at which was achieved the maximum strength,  $\phi = 0.600$ .

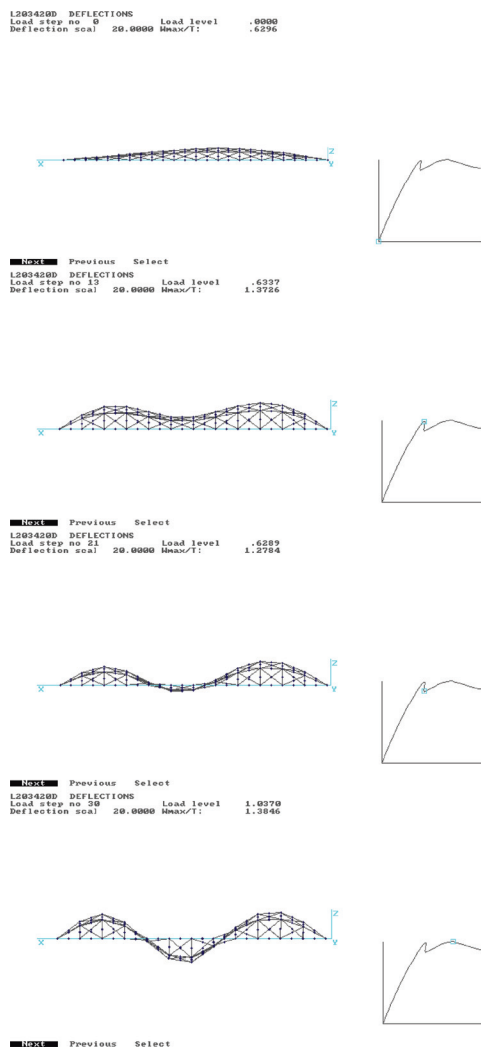


Figure 7. Evolution of deformed shape of plate D until collapse. On the right side it is indicated the corresponding loading point on the stress-strain curve.

This value is well above the value obtained for the plate H,  $\phi = 0.527$ . It can be concluded that this low value should not be considered as the probability of finding plates with reduced and perfect initial imperfections in ternary mode is practically nil and any disturbance in the geometry leads to a different failure mode. Moreover, it was confirmed on the plate K having only very small initial imperfections in the ternary mode, where the asymmetry generated by the numerical truncation was enough to make evolve the failure mode to the critical one.

The strength degradation from a ternary failure mode occurred on the plates D and H, raises the



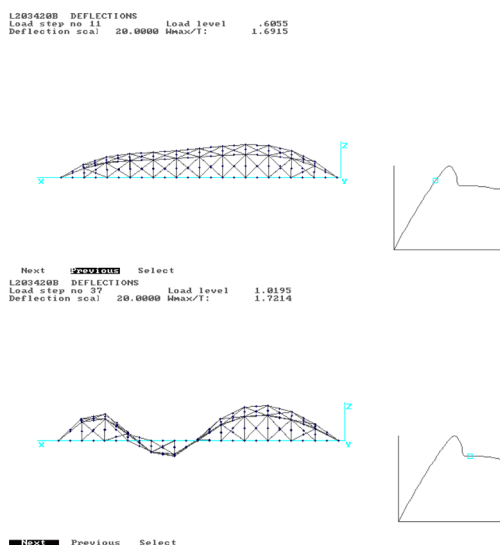


Figure 8. Evolution of deformed shape of plate B until collapse. On the right side it is indicated the corresponding loading point on the stress-strain curve.

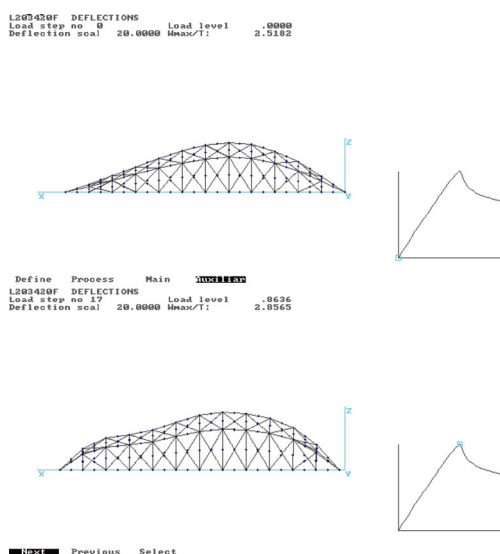


Figure 9. Evolution of deformed shape of plate F until collapse. On the right side it is indicated the corresponding loading point on the stress-strain curve.

question of the conditions under which this degradation occurs. It is resulting from the initial amplitude of the ternary mode or deformed amplitude generated during the deformation under loading. The maximum strength of plate I confirms this conclusion because the existence of an important ternary component leads to a collapse with  $m = 3$

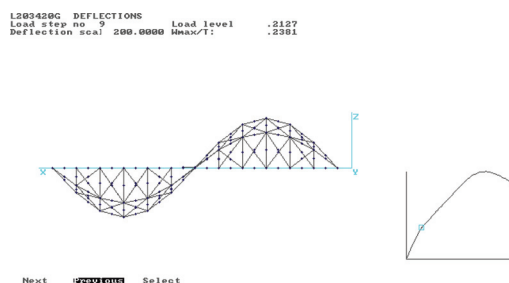


Figure 10. Deformed shape of plate G before collapse. On the right side it is indicated the corresponding loading point on the stress-strain curve.

and very low resistance in spite of the existence of a component in  $m = 2$ . In the plate J it was maintained the amplitude of distortions but reversing the modes and, as result, it was obtained a different failure mode with a much higher strength, +16%. However, the strength of this plate (J) is 4.5% lower than the plate with the same amplitude but where distortion of the third component is not present, plate A.

One can conclude that the presence of a considerable ternary component degrades the strength of the plates with aspect ratio equal to 2. Eventually, similar conclusion can be applied to other aspect ratios due to higher order modes than the critical one induce a more pronounced local collapse.

Once clarified the extreme importance of the failure mode on the ultimate strength it was analysed a set of plates allowing to define the ratio of amplitudes of the distortions which promote a change of the mode of collapse to the critical one. This set of plates as a maximum amplitude of distortions according to the expression Guedes Soares (1988), and varying the ratio between the first and second components.

Figure 11 presents the behaviour of these plates. It is evident that there is a different behaviour as the ratio  $a_{11}/a_{21}$  increases. When this ratio is above 3 the plate does not develop a critical failure mode and denotes high strength levels as shown in Figure 12 and a complex failure mode.

For plate R with ratio between the two components equal to 3, the situation is intermediate noting a sharp collapse on mode  $m = 2$ , but, however, one of the half-waves have a length much lower than the other increasing the maximum deformations in the area corresponding to shortest half-wave.

The tangent modulus of this plate is similar to the one of plates Q and B until the average stress approaches the minimum critical stress. At this point the tangent modulus is dramatically reduced due to the development of deformations on the critical mode and the stress-end shortening curve

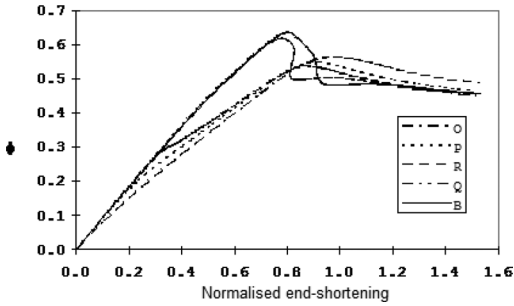


Figure 11. Normalized stress-end-shortening curves of simply supported restrained plates with  $\alpha = 2$ ,  $\beta = 3.38$ , average distortions and different ratio between the first and second component of distortions, respectively 1(O), 2(P), 3(R), 4(Q) e 6(B).

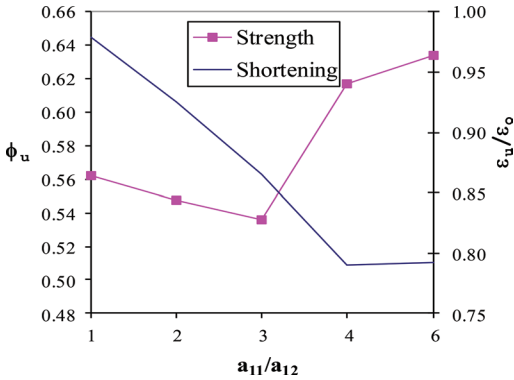


Figure 12. Dependence of ultimate strength and end-shortening for plates with  $\alpha = 2$ ,  $\beta = 3.38$  and average amplitude of distortions on the ratio between the first and second mode.

approaches the curves of the plates O and P which have a marked critical mode of collapse.

Regarding the behaviour in the post collapse regime it is to point out the similarity in behaviour between all plates. The loss of strength with increasing plastic strain is small, particularly for plates with a critical failure mode. The remaining plates, Q and B have a greater reduction since they have greater ultimate strength, but from a shortening of 1.2 they follow the common curve.

Finally, it is analysed the variation of ultimate end-shortening with the ratio between the Fourier components of initial imperfections presented in Figure 12.

Slender plates having a dominant primary mode ( $m = 1$ ) and average or high levels of initial imperfections present an ultimate normalised end-shortening less than one and it tends to reduce when the ratio between the first and second components

of imperfections increases but keeping the amplitude constant. Minimum values of ultimate end-shortening are found for this mode and the plates present a rigidity close to the Young modulus of material until collapse. This rigidity of plates having a dominant primary mode is much higher than the one of critical plates, as shown in Figure 13.

Figure 14 shows that the ultimate end-shortening increases with the amplitude of imperfections almost linearly for each mode of collapse. Highest

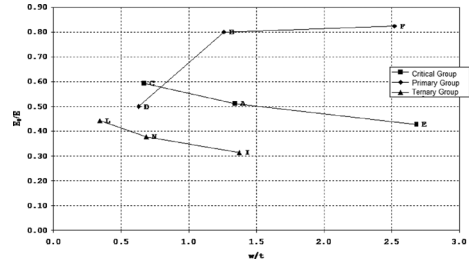


Figure 13. Dependence of normalised secant modulus at collapse on shape and amplitude of initial imperfections (ratio between components of 6).

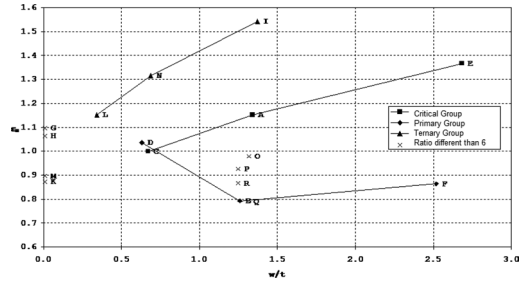


Figure 14. Dependence of normalised end-shortening at collapse on shape and amplitude of initial imperfections (ratio between components of 6).

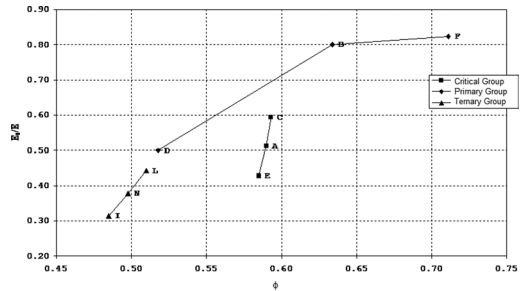


Figure 15. Relation between the normalised secant modulus and the ultimate strength on slender plates with aspect ratio of 2.

values are found for collapse on the ternary mode. Plate I has the highest value to which correspond the lowest ultimate strength, 0.485.

Figure 15 presents the relation between the ultimate strength and the normalised secant modulus at collapse. It may be concluded that there is a direct relation between both especially for each mode of collapse. Plates with initial critical mode does not show a great variation on the ultimate strength despite a decrease on the secant modulus with increasing amplitude of imperfections.

### 3 CONCLUSIONS

The collapse mode is a very important factor determining the strength of slender plates. The failure mode is due to two initial conditions, namely, the maximum amplitude and the relative amplitude of different modes of initial imperfections. The second condition is more important than the first one. The aforementioned initial geometry affects very much the behaviour of the plates during the pre-collapse but the post collapse region is not very sensitive to initial conditions, which may be explained by the development of large deformations in all cases which make irrelevant the effects of the initials conditions. On the other hand, the collapse is almost always localised so initial conditions will not affect much the post collapse behaviour.

The strength variations found for slender plates are high, between 0.711 and 0.485 with a mean value of 0.572 and a COV of 9.9%, so one can say that it is necessary to take into account the shape of imperfection on the prediction of restraint plate's strength. To do this it is imperative to determine accurately the modes and amplitudes of these imperfections in real plates and consider the most likely initial mode.

In thick plates it must be considered also as very important the maximum amplitude of the imperfections, despite the predominant initial mode strongly influence the maximum resistance level reached.

The degradation of ultimate strength of restrained plates due to the amplitude imperfections is much larger in thick plates than in slender plates. It should be noted however that the maximum amplitude normalised by the thickness is usually smaller in thick plates than slender.

The presence of a considerable ternary component degrades the strength of the plates with aspect ratio equal to 2.

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